CES 544 Project 3
Space Time Block Coding

I. Objectives

1. Understand the concepts of transmit diversity and receiver diversity.
2. Understand the procedure of orthogonal space time block coding.
3. Learn to simulate a communication system with space time block coding.

II. Theories

Orthogonal space time block code (STBC) can be applied to communication system with 2 transmit antennas and *M* receive antennas. It can achieve the full diversity order of 2M without the knowledge of the channel state information at the communication transmitter.

The operation of STBC is illustrated in the following figure.

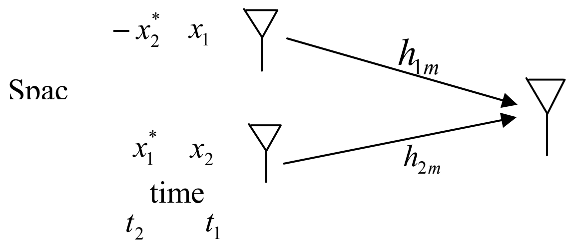

Fig. 1 System structure of STBC

At the first symbol period, the symbol x_1 is transmitted at the first transmit antenna and the symbol x_2 is transmitted at the second transmit antenna. At the next symbol period, the symbol $-x_2^*$ is transmitted at the first transmit antenna, and the symbol x_1^* is transmitted at the second transmit antenna. If we assume that the channel keeps constant during two consecutive symbol periods, then the system equation between the two transmit antennas and the m-th receive antenna can be expressed as

$$\begin{bmatrix} y_{1m} \\ y_{2m}^* \end{bmatrix} = \begin{bmatrix} h_{1m} & h_{2m} \\ h_{2m}^* & -h_{1m}^* \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_{1m} \\ n_{2m}^* \end{bmatrix},$$

or in matrix format

where h_{nm} is the channel impulse response between the n-th Tx antenna and m-th Rx antenna, y_{km} is the received symbol on the m-th Rx antenna at the k-th symbol period, for k = 1, 2, and n_{km} is the AWGN at the m-th Rx antenna during the k-th symbol period.

Stacking the system equations of all M Rx antennas, we have the system equation of the 2xM system as

$$\begin{bmatrix} \mathbf{y}_1 \\ \vdots \\ \mathbf{y}_M \end{bmatrix} = \begin{bmatrix} \mathbf{H}_1 \\ \vdots \\ \mathbf{H}_M \end{bmatrix} \mathbf{x} + \begin{bmatrix} \mathbf{n}_1 \\ \vdots \\ \mathbf{n}_M \end{bmatrix}$$

or in compact form

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} .$$

At the receiver, performing the following operation

$$\mathbf{r} = \mathbf{H}^H \mathbf{y} = \mathbf{H}^H \mathbf{H} x + \mathbf{H}^H \mathbf{n} = \sum_{m=1}^{M} \mathbf{H}_m^H \mathbf{H}_m \mathbf{x} + \sum_{m=1}^{M} \mathbf{H}_m^H \mathbf{n}_m$$

which can be alternatively represented as

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} \sum_{m=1}^{M} \left[| h_{1m} |^2 + | h_{1m} |^2 \right] & 0 \\ 0 & \sum_{m=1}^{M} \left[| h_{1m} |^2 + | h_{1m} |^2 \right] \end{bmatrix} \begin{bmatrix} x_1 \\ x_2^* \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

where z_1 and z_2 are noise components. The symbols x_1 and x_2 can be detected by performing demodulation over $r_1 / \sum_{m=1}^{M} \left[| h_{1m} |^2 + | h_{1m} |^2 \right]$ and $r_2^* / \sum_{m=1}^{M} \left[| h_{1m} |^2 + | h_{1m} |^2 \right]$, respectively.

III. Procedures

1. First we are going to examine the performance of BPSK modulated system WITHOUT diversity.
   (1) Following the procedures in Project 2, set up a BPSK modulated system with Rayleigh fading and AWGN. To simplify the system construction, let each slot has two symbols.
   (2) Set the simulation parameters to: fd = 10 Hz, symbol rate = 24.3 ksym/sec. Find the BER curve of the BPSK system without diversity in the range of Eb/N0 = [0:5:25] dB. At each value of Eb/N0, simulate at least 40,000 slots (each slot has 2 symbols).
   (3) Compare your simulation results with the theoretical results described by the following equation (plot them in the same figure)

$$P_{BPSK} = \frac{1}{2} - \frac{1}{2}\sqrt{\frac{\gamma}{1+\gamma}}$$

   where P_{BPSK} is the BER, and $\gamma = E_b / N_0$ (linear value). Your simulation results should match the theoretical results very well. If they don't match, go back to debug your simulation program. (Debug hint: 1. if you set noise_amplitude = 0, the BER should always be zero. 2. if you still couldn't find the problem, set noise_amplitude = 0, and fading = 1. Again, the BER should always be zero.).

2. Simulation of a system with 2 Tx antennas and 1 Rx antenna.
   (1) Extend the BPSK system designed in step 1 to a system with two Tx antennas.
   (This can be done by using the following sample statement.

   data_channel_1 =1/sqrt(2) data_1.*fading_1;*

data_channel_2 =1/sqrt(2) data_2.*fading_2;*
data_rx = data_channel_1 + data_channel_2 + noise;

where data_n is the STBC encoded contents to be transmitted on the n-th antenna, and data_rx is the contents observed by the receive antenna. The term 1/sqrt(2) is used to make sure that the total power on the two transmit antennas is normalized to 1.)

(2) Implement STBC at both Tx and Rx by following the procedure described in Section II.
Hint 1: Some useful matlab operators: conj(a): the conjugate of a; A': the Hermitian (transpose and conjugate) of matrix A; A.': the transpose of matrix A.
Hint 2: data_1 = [x(1), -conj(x(2))]; % the symbols at the 1st Tx antenna
Hint 3: y = [data_rx(1); conj(data_rx(2))]
Hint 4: h11 = fading_1(1), h21 = fading_2(1). Use these values to formulate the matrix H.
Hint 5: Let r = H'*y. Performing demodulation over r(1)/(abs(h11)^2 + abs(h21)^2), and conj(r(2))/ (abs(h11)^2 + abs(h21)^2), respectively.

(3) Find the BER performance of the system at EbN0 = [0:5:20] dB. Use at least 40,000 slots for each point on the BER curve. Plot the BER curve in the same figure as the BER performance of the system without diversity.

3. (Bonus) Simulation of a system with 2 Tx antennas and 2 Rx antennas.

(1) Extend the BPSK system designed in step 2 to a system with two Rx antennas.
(This can be done by using the following sample statement.
% data sent out by the first Tx antenna
*data_channel_11 = 1/sqrt(2)*data_1.*fading_11;*
*data_channel_12 = 1/sqrt(2)*data_1.*fading_12;*
% data sent out by the second Tx antenna
*data_channel_21 = 1/sqrt(2)*data_2.*fading_21;*
*data_channel_22 = 1/sqrt(2)*data_2.*fading_22;*
% data at the first Rx antenna
data_rx_1 = data_channel_11 + data_channel_21 + noise_1;
% data at the second Rx antenna
data_rx_2 = data_channel_12 + data_channel_22 + noise_2;

where data_n is the contents to be transmitted on the n-th antenna, and data_rx_m is the contents observed by the m-th receive antenna. It should be noted that we have 4 different channels (fadings) in this case.)

(2) Implement the STBC decoding algorithm described in Section II.

(3) Find the BER performance of the system at EbN0 = [0:5:20] dB. Use at least 40,000 slots for each point on the BER curve. Plot the BER curve in the same figure as the BER performance of the system without diversity.