Optimal Diversity Combining based on Noisy Channel Estimation

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Abstract— **The performances of coherent diversity receivers with noisy channel estimation are examined. Fading channel gain estimates are modeled as sums of the true fading channel gain values plus independent Gaussian distributed estimation errors. The optimal diversity receiver for coherent reception with noisy channel state information and independent and identically distributed fading channels is derived. Exact expressions for the average error probability of optimal diversity MPSK with noisy channel estimation are derived for Rayleigh and Ricean fading channels; closed-form expressions are obtained for some special cases. Some interesting observations regarding practical diversity receiver design for higher-order modulation formats are drawn.**

I. INTRODUCTION

Diversity reception is a classical method used in wireless communication systems for combating the hostile nature of fading channels, and the error performance analysis of diversity receivers in fading channels has been a field of long-time interest, see [1]-[7] and the references therein. A commonly used method for analyzing error probability of digital communication systems is to average the conditional error probability (CEP) $P(E|\gamma)$ over the receiver singal-to-noise ratio (SNR) γ, with the help of the probability density function (pdf) [2], characteristic function (CHF) [3], or moment generating function (MGF) [4], [5] of γ.

Most previous works about performance analyses of coherent diversity systems assume that the receiver has perfect knowledge (noiseless estimation) of the fading channels. In the literature, only few works are devoted to the performance analyses of non-ideal systems. In [8], the effect of Gaussian error in maximal ratio combining is studied. However, the mathematical models assumed preclude using the analysis for independent additive noise and the analysis is valid only for Gaussian error originating from temporal decorrelation [9]. Further, digital modulations and error probability are not considered in [8]. The error probabilities of systems with non-ideal channel information for *non-diversity* systems are obtained in [10] and [11] by seeking the pdf of the equivalent output noise at the receiver, which is usually non-Gaussian distributed and extremely complicated for analyses.

In this paper, error performances of optimal coherent diversity receivers operating on independent and identically distributed (i.i.d.) fading channels with noisy channel estimation are analyzed. It is shown that the conventional MRC receiver is no longer optimal when there is channel estimation error in the system. A new optimal decision rule for coherent diversity receivers with noisy channel estimation is proposed to minimize the error probability of the system. Based on this decision rule, the error probabilities for optimal coherent diversity receivers are derived for MPSK systems in both Rayleigh and Ricean fading channels. The error probability in Rayleigh fading channels is evaluated with the help of the MGF of the power of the fading channel, and a complex Gaussian distribution based functional equivalency is employed for the evaluation of system error performance in Ricean fading channels. Simulation results are in excellent agreement with the theoretical results.

The rest of this paper is organized as follows. The system models are given in Section II. Section III derives a new optimal decision rule for diversity receivers operating with noisy estimates of i.i.d. fading channels. The error probabilities of the corresponding receivers in Rayleigh and Ricean fading channels are derived in Section IV. Numerical examples are given in Section V, and Section VI concludes the paper.

II. SYSTEM AND CHANNEL MODELS

We consider a communication system with N diversity receivers. After sampling at the receiver, the input-output relationship of the equivalent baseband system can be written in matrix form as

$$\mathbf{y}_k = \mathbf{h}_k \cdot x_k + \mathbf{z}_k \tag{1}$$

where $\mathbf{y}_k = [y_1(k), y_2(k), \cdots, y_N(k)]^T \in \mathbb{C}^{N\times 1}$ is the sampled output of the receivers with $\mathbf{A}^T$ representing the transpose of matrix $\mathbf{A}$, $\mathbf{h}_k = [h_1(k), h_2(k), \cdots, h_N(k)]^T \in \mathbb{C}^{N\times 1}$ is the equivalent discrete-time channel gain (CG) vector of the physical time-varying fading channels, x_k is the MPSK modulated symbol transmitted at time instant k, and $\mathbf{z}_k = [z_1(k), z_2(k), \cdots, z_N(k)]^T \in \mathbb{C}^{N\times 1}$ is a zero-mean additive white Gaussian noise vector with covariance matrix $N_0\mathbf{I}_N$, and $\mathbf{I}_N$ is the $N \times N$ identity matrix.

For Rayleigh and Ricean fading channels, the discrete-time CG vector $\mathbf{h}_k$ is made up of complex Gaussian random variables (CGRVs) with mean vector $\mathbf{u}$ and covariance matrix $\mathbf{R}_{hh}$, *i.e.*, $\mathbf{h}_k \sim \mathcal{N}(\mathbf{u}, \mathbf{R}_{hh})$. The variance $\sigma^2_{h_n}$, power Ω_n, and mean value u_n of $h_n(k)$ have the following relationships,

$$|u_n| = \sqrt{\frac{K\Omega_n}{K+1}} = \sqrt{K\sigma^2_{h_n}} \tag{2}$$

where K is the Ricean factor defined as the ratio of the powers of the specular component and the scattering components of the fading channel. One has $K = 0$ for a Rayleigh fading channel, and, thus, $\mathbf{u} = \mathbf{0}$.

The receiver performs coherent detection of the received samples based on the estimated CG vector $\hat{\mathbf{h}}_k = [\hat{h}_1(k), \hat{h}_2(k), \cdots, \hat{h}_N(k)]^T \in \mathbb{C}^{N\times 1}$. The estimated CG vector $\hat{\mathbf{h}}_k$ is modeled as the sum of the true CG vector $\mathbf{h}_k$ and the estimation error vector $\mathbf{e}_k = [e_1(k), e_2(k), \cdots, e_N(k)]^T$ as

$$\hat{\mathbf{h}}_k = \mathbf{h}_k + \mathbf{e}_k \tag{3}$$

where the elements of the error vector $\mathbf{e}_k$ are assumed to be independent zero-mean CGRVs, and they are independent of the elements of $\mathbf{h}_k$. Then the covariance matrices $\mathbf{R}_{h\hat{h}} = E(\mathbf{h}_k^H \hat{\mathbf{h}}_k)$ and $\mathbf{R}_{\hat{h}\hat{h}} = E(\hat{\mathbf{h}}_k^H \hat{\mathbf{h}}_k)$ can be computed as

$$\mathbf{R}_{h\hat{h}} = E[\mathbf{h}_k(\mathbf{h}_k + \mathbf{e}_k)^H] = \mathbf{R}_{hh} \quad (4a)$$

$$\mathbf{R}_{\hat{h}\hat{h}} = E[(\mathbf{h}_k + \mathbf{e}_k)(\mathbf{h}_k + \mathbf{e}_k)^H] = \mathbf{R}_{hh} + \mathbf{R}_{ee} \quad (4b)$$

where $\mathbf{R}_{ee} = \text{diag}(\sigma_{e_1}^2, \sigma_{e_2}^2, \cdots, \sigma_{e_N}^2)$ is an $N \times N$ diagonal matrix with $\sigma_{e_n}^2 = E[|e_n(k)|^2]$ being the power (variance) of the channel estimation error $e_n(k)$. Based on these definitions, we have the following proposition.

Proposition 1: The estimated CG vector $\hat{\mathbf{h}}_k$ and the true CG vector $\mathbf{h}_k$ are jointly Gaussian distributed, and the conditional pdf $p(\mathbf{h}_k|\hat{\mathbf{h}}_k)$ can be written as

$$p(\mathbf{h}_k|\hat{\mathbf{h}}_k) = \frac{1}{\det(\pi\mathbf{R}_{h|\hat{h}})} \exp\left[-(\mathbf{h}_k - \mathbf{u}_{h|\hat{h}})^H \mathbf{R}_{h|\hat{h}}^{-1} (\mathbf{h}_k - \mathbf{u}_{h|\hat{h}})\right] \quad (5)$$

where

$$\mathbf{u}_{h|\hat{h}} = \mathbf{u} + \mathbf{R}_{hh}(\mathbf{R}_{hh} + \mathbf{R}_{ee})^{-1}(\hat{\mathbf{h}}_k - \mathbf{u}) \quad (6a)$$

$$\mathbf{R}_{h|\hat{h}} = \mathbf{R}_{hh} - \mathbf{R}_{hh}(\mathbf{R}_{hh} + \mathbf{R}_{ee})^{-1}\mathbf{R}_{hh} \quad (6b)$$

is the conditional mean vector and conditional covariance matrix, respectively.

Proof: The proof of Proposition 1 is omitted here for the sake of brevity. ■

To facilitate analysis, we define the covariance coefficient between the estimated CG $\hat{h}_n(k)$ and the true CG $h_n(k)$ of the nth sub-channel as

$$\rho_n = \frac{E\{[h_n(k) - u][\hat{h}_n(k) - u]^*\}}{\sqrt{\sigma_{h_n}^2 \sigma_{\hat{h}_n}^2}} = \sqrt{\frac{\sigma_{h_n}^2}{\sigma_{h_n}^2 + \sigma_{e_n}^2}} \quad (7)$$

where a^* denotes the complex conjugate of the complex number a, $\sigma_{h_n}^2 = E[|h_n(k) - u|^2]$ and $\sigma_{\hat{h}_n}^2 = E[|\hat{h}_n(k) - u|^2] = \sigma_{h_n}^2 + \sigma_{e_n}^2$ is the variance of $h_n(k)$ and $\hat{h}_n(k)$, respectively. The value of ρ_n is in the interval $(0, 1]$ with $\rho_n = 1$ corresponding to noiseless (perfect) channel information at the receiver. Since diversity receivers usually use the same channel estimation algorithm for all the branches, we assume that $\rho = \rho_1 = \rho_2 = \cdots = \rho_N$ for systems with i.i.d. fading channels, and the coefficient ρ is assumed to be known to the receiver once the channel estimation algorithm is chosen.

III. Optimal Diversity Receiver with Noisy Channel Estimation

In this section, an optimal decision rule for coherent diversity reception is proposed to minimize the error probability of systems with noisy channel estimation.

Theorem 1: For diversity receivers with noisy estimation of i.i.d. fading channels, if the transmitted symbols are equiprobable, then the detection rule that minimizes the system error probability is

$$\hat{x}_k = \underset{s_m \in \mathcal{S}}{\text{argmin}} \{|\alpha_k - s_m|^2\} \quad (8)$$

where $\mathcal{S} = \{s_m = \sqrt{E_s}e^{-j2\pi\frac{m}{M}} | m = 1, 2, \cdots, M\}$ is the modulation alphabet set, and $\alpha_k = [\rho^2\hat{\mathbf{h}}_k + (1-\rho^2)\mathbf{u}]^H \mathbf{y}_k$ is the decision variable.

Proof: From Proposition 1, we know that $\mathbf{h}_k$ conditioned on $\hat{\mathbf{h}}_k$ is Gaussian distributed; it follows from (1) that $\mathbf{y}_k$ conditioned on $\hat{\mathbf{h}}_k$ and x_k is also Gaussian distributed. If we assume that the symbol $s_m \in \mathcal{S}$ is transmitted, where $\mathcal{S}$ is the modulation alphabet set, then we have $\mathbf{y}_k|(\hat{\mathbf{h}}_k, s_m) \sim \mathcal{N}(\mathbf{u}_{y|\hat{h},s_m}, \mathbf{R}_{y|\hat{h},s_m})$. Combining (1), (6), (7) and the fact that $\mathbf{R}_{hh} = \sigma_h^2 \mathbf{I}_N$ for i.i.d. fading channels, one can compute the conditional mean vector $\mathbf{u}_{y|\hat{h},s_m}$ and conditional covariance matrix $\mathbf{R}_{y|\hat{h},s_m}$ as

$$\mathbf{u}_{y|\hat{h},s_m} = [\rho^2\hat{\mathbf{h}}_k + (1-\rho^2)\mathbf{u}]s_m \quad (9a)$$

$$\mathbf{R}_{y|\hat{h},s_m} = (\rho^2\sigma_e^2 E_s + N_0)\mathbf{I}_N. \quad (9b)$$

With the help of (9), we employ the maximum *a posteriori* (MAP) detection rule to the conditionally Gaussian distributed variable $\mathbf{y}_k|(\hat{\mathbf{h}}_k, s_m)$, and the optimal decision rule that minimizes the system error probability is,

$$\hat{x} = \underset{s_m \in \mathcal{S}}{\text{argmin}} \{\|\mathbf{y}_k - [\rho^2\hat{\mathbf{h}}_k + (1-\rho^2)\mathbf{u}]s_m\|^2\}. \quad (10)$$

To simplify the representation of the decision rule given in (10), we let $\mathbf{d}_k = \rho^2\hat{\mathbf{h}}_k + (1-\rho^2)\mathbf{u}$. Expanding (10), one can get the following equivalent decision rule

$$\hat{x}_k = \underset{s_m \in \mathcal{S}}{\text{argmin}} \{-2\Re(\mathbf{d}_k^H \mathbf{y}_k s_m)\} = \underset{s_m \in \mathcal{S}}{\text{argmin}} \{-2\Re(\alpha_k s_m)\}. \quad (11)$$

After some algebraic manipulations, one can show that (11) is equivalent to (8), and this completes the proof. ■

If the receiver has perfect knowledge of the fading channel, *i.e.*, $\rho = 1$, then the decision variable becomes $\alpha_k = \hat{\mathbf{h}}_k^H \mathbf{y}_k$, and the decision rule specializes to the conventional MRC diversity receiver. However, when $\rho < 1$, one can see from the decision rule given in (8) that the conventional MRC receiver is not optimal in the presence of channel estimation error.

IV. Error Performance Analyses

A. Conditional Error Probability

We first evaluate the conditional error probability (CEP) $P(E|\hat{\mathbf{h}}_k)$, which will be used to obtain the error probability of the diversity system in Rayleigh and Ricean fading channels.

It can be seen from the decision rule of (8) that the detected symbol $\hat{x}_k$ should have the smallest Euclidean distance from the decision variable α_k. Based on this decision rule, the detection region for α_k of the MPSK symbol s_m should be a $\frac{2\pi}{M}$ angle sector centered around s_m as shown in Fig. 1, and the conditional error probability $P(E|\hat{\mathbf{h}}_k, s_m)$ equals the probability that α_k is outside of the detection region of s_m when s_m is transmitted.

Since the received sample vector $\mathbf{y}_k$ conditioned on $\hat{\mathbf{h}}_k$ is Gaussian distributed, the decision variable α_k conditioned on $\hat{\mathbf{h}}_k$ is also Gaussian distributed, $\alpha_k|(\hat{\mathbf{h}}_k, s_m) \sim \mathcal{N}(u_{\alpha|\hat{h},s_m}, \sigma_{\alpha|\hat{h},s_m}^2)$, with the conditional mean and conditional variance given by

$$u_{\alpha|\hat{h},s_m} = \|\mathbf{d}_k\|^2 s_m \quad (12a)$$

$$\sigma_{\alpha|\hat{h},s_m}^2 = \|\mathbf{d}_k\|^2(\rho^2\sigma_e^2 E_s + N_0) \quad (12b)$$

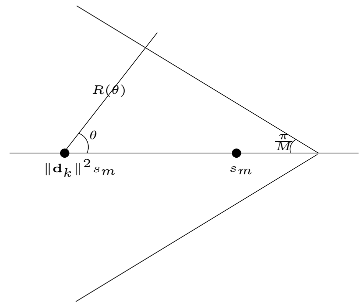

Fig. 1. The decision region for MPSK modulation.

with $\mathbf{d}_k = \rho^2\hat{\mathbf{h}}_k + (1-\rho^2)\mathbf{u}$. To simplify the derivations of the CEP, we represent the conditional pdf $p(\alpha_k|\hat{\mathbf{h}}_k, s_m)$ in a polar coordinate system with origin at $u_{\alpha|\hat{h},s_m} = \|\mathbf{d}_k\|^2 s_m$, and the corresponding pdf written in the polar coordinate system is

$$p(r,\theta|\hat{\mathbf{h}}_k, s_m) = \frac{r}{\pi\sigma^2_{\alpha|\hat{h},s_m}} \exp\left(-\frac{r^2}{\sigma^2_{\alpha|\hat{h},s_m}}\right). \qquad (13)$$

With (13) and the decision region shown in Fig. 1, the CEP $P(E|\hat{\mathbf{h}}_k)$ can be computed as

$$\begin{aligned} P(E|\hat{\mathbf{h}}_k) &= 2\sum_{m=1}^{M} P(s_m)\int_0^{\pi-\frac{\pi}{M}}\int_{R(\theta)}^{+\infty} p(r,\theta|\hat{\mathbf{h}}_k, s_m)drd\theta \\ &= \frac{1}{\pi}\int_0^{\pi-\frac{\pi}{M}} \exp\left\{-\frac{\|\mathbf{d}_k\|^4 E_s \sin^2(\frac{\pi}{M})}{\sigma^2_{\alpha|\hat{h},s_m}\sin^2(\phi)}\right\} d\phi \qquad (14) \end{aligned}$$

where $R(\theta) = \frac{\|\mathbf{d}_k\|^2|s_m|sin(\pi/M)}{\sin(\theta+\pi/M)}$, $P(s_m) = \frac{1}{M}$ for equiprobable transmitted symbols. If we define the average SNR γ_n as

$$\gamma_n = \frac{\Omega_n E_s}{N_0} = \frac{(K+1)\sigma^2_{h_n}E_s}{N_0} \qquad (15)$$

where Ω_n is the power of the nth fading channel, then the CEP can be written in the form of (16) at the top of the next page, where $\sigma^2_{\hat{h}_n}$ is the variance of the estimated CG $\hat{h}_n(k)$, and the identity $\rho^2 = \sigma^2_{h_n}/\sigma^2_{\hat{h}_n}$ from (7) has been used.

B. Error Probability in Rayleigh Fading Channels

For Rayleigh fading channels, the Ricean factor $K = 0$, and both the true CG $\mathbf{h}_k$ and the estimated CG $\hat{\mathbf{h}}_k$ are zero-mean CGRVs, *i.e.*, $\mathbf{u} = 0$. The CEP described in (16) for Rayleigh fading channels can be simplified to

$$P(E|\hat{\mathbf{h}}_k) = \frac{1}{\pi}\int_0^{\pi-\frac{\pi}{M}}\prod_{n=1}^{N}\exp\left\{-\tilde{\gamma}_{ray}\frac{|\hat{h}_n(k)|^2\sin^2(\frac{\pi}{M})}{\sigma^2_{\hat{h}_n}\sin^2(\phi)}\right\}d\phi \quad (17)$$

where

$$\tilde{\gamma}_{ray} = \frac{\rho^2}{\gamma_n(1-\rho^2)+1}\gamma_n \qquad (18)$$

is the equivalent SNR for systems with channel estimation error, and is obtained from scaling the average SNR γ_n by a factor $\beta = \frac{\rho^2}{\gamma_n(1-\rho^2)+1}$. Based on the fact that $0 < \rho \le 1$, it can be easily shown that $\tilde{\gamma}_{ray} \le \gamma_n$, and equality holds when $\rho = 1$.

If we let $g_n = |\hat{h}_n(k)|^2$, then the random variable g_n is χ^2-distributed with 2 degrees of freedom and the unconditional error probability can be directly evaluated with the MGF method. The MGF of the χ^2-distributed random variable g_n is [4, p. 19]

$$\Phi_g(s) = E(e^{sg_n}) = (1 - s\sigma^2_{\hat{h}_n})^{-1} \qquad (19)$$

where $\sigma^2_{\hat{h}_n} = E(|\hat{h}_n(k)|^2) = E(g_n)$ is the variance of the estimated Rayleigh fading channel. Using the identity presented in (19), the unconditional error probability $P(E) = E[P(E|\hat{\mathbf{h}}_k)]$ for i.i.d. Rayleigh fading channels can be computed as

$$P(E) = \frac{1}{\pi}\int_0^{\pi-\frac{\pi}{M}}\prod_{n=1}^{N}\left[1+\tilde{\gamma}_{ray}\frac{\sin^2(\frac{\pi}{M})}{\sin^2(\phi)}\right]^{-1} d\phi. \qquad (20)$$

Note that the result in (20) agrees with [6, eqn. (24)] for the special case of $\rho = 1$, corresponding to the case when the receiver has perfect knowledge of the fading channel.

For communication systems with BPSK modulation, we have $M = 2$, and the error probability of the diversity receiver can be written in closed-form [14, eqn. (3.259.3)],

$$P(E) = \frac{\Gamma(N+\frac{1}{2})}{2\sqrt{\pi}N!(\gamma_{ray}+1)^N}\ {}_2F_1(N,\frac{1}{2};N+1;\frac{1}{\gamma_{ray}+1}) \quad (21)$$

where $\Gamma(x)$ is the Gamma function, and ${}_2F_1(\cdot)$ is the Gauss hypergeometric function. When there is no diversity in the system, *i.e.* $N = 1$, the error probability (20) for the MPSK system can be expressed in closed-form by changing the variable of integration to $z = \cot(\phi)$,

$$\begin{aligned} P(E) &= \frac{M-1}{M} - \sqrt{\frac{\tilde{\gamma}_{ray}\sin^2(\frac{\pi}{M})}{1+\tilde{\gamma}_{ray}\sin^2(\frac{\pi}{M})}} \times \\ &\left[\frac{1}{2}+\frac{1}{\pi}\arctan\left(\sqrt{\frac{\tilde{\gamma}_{ray}\sin^2(\frac{\pi}{M})}{1+\tilde{\gamma}_{ray}\sin^2(\frac{\pi}{M})}}\cot\left(\frac{\pi}{M}\right)\right)\right] (22) \end{aligned}$$

For the special case of perfect channel information, we have $\tilde{\gamma}_{ray} = \gamma_n$, and (22) agrees with the result previously obtained in [11, eqn. (36)] through a different approach.

For diversity systems with $M > 2$, the symbol error rate given in (20) must be evaluated numerically. The expression for the SER in (20) contains one integration with finite limits, and the integrand is constituted of only elementary functions. Thus, it can be easily evaluated with simple numerical methods.

C. Error Probability in Ricean Fading Channels

The unconditional error probability in Ricean fading channels is derived in this subsection.

For i.i.d. fading channels, the pdf of the estimated CG $\hat{\mathbf{h}}_k$ is

$$p(\hat{\mathbf{h}}_k) = \prod_{n=1}^{N}\frac{1}{\pi\sigma^2_{\hat{h}_n}}\exp\left[-\frac{|\hat{h}_n(k)-u_n|^2}{\sigma^2_{\hat{h}_n}}\right]. \qquad (23)$$

Combining (16) with (23), we obtain the unconditional error probability $P(E)=\int_{\{\hat{\mathbf{h}}_k\}}P(E|\hat{\mathbf{h}}_k)p(\hat{\mathbf{h}}_k)d\hat{\mathbf{h}}_k$ in a Ricean fading channel as

$$P(E) = \frac{1}{\pi}\int_0^{\pi-\frac{\pi}{M}}\prod_{n=1}^{N}\lambda_n(\phi)d\phi \qquad (24)$$

$$P(E|\hat{\mathbf{h}}_k) \quad = \quad \frac{1}{\pi}\int_0^{\pi-\frac{\pi}{M}} \prod_{n=1}^{N} \exp\left\{-\frac{\rho^2\gamma_n|\hat{h}_n(k) - u_n(1-\frac{1}{\rho^2})|^2 \sin^2(\frac{\pi}{M})}{\sigma^2_{\hat{h}_n}[\gamma_n(1-\rho^2)+K+1]\sin^2(\phi)}\right\} d\phi \tag{16}$$

where

$$\lambda_n(\phi) = \frac{1}{\pi\sigma^2_{\hat{h}_n}}\int_{\{\hat{h}_n\}} \exp\left[-\frac{g|\hat{h}_n - au_n|^2 + |\hat{h}_n - u_n|^2}{\sigma^2_{\hat{h}_n}}\right] d\hat{h}_n, \tag{25}$$

with g, a and the equivalent SNR $\tilde{\gamma}_{rice}$ for Ricean fading channel being defined as

$$g = \frac{\tilde{\gamma}_{rice}\sin^2(\frac{\pi}{M})}{\sin^2(\phi)}, \tag{26a}$$

$$a = (1-\frac{1}{\rho^2}), \tag{26b}$$

$$\tilde{\gamma}_{rice} = \frac{\rho^2\gamma_n}{\gamma_n(1-\rho^2)+K+1}. \tag{26c}$$

Since the integrand of (25) is an exponential function of the square of the integration variable $\hat{h}_n(k)$, we can write it as the product of a Gaussian pdf and a constant term. Then, using the properties of Gaussian pdfs, one can get the closed-form solution of $\lambda_n(\phi)$ as

$$\begin{aligned}\lambda_n(\phi) &= \frac{1}{g+1}\exp\left[-\frac{g(a-1)^2}{(g+1)\sigma^2_{\hat{h}_n}}|u_n|^2\right] \times \\ &\quad \int_{\{\hat{h}_n\}} \frac{1}{\pi\sigma^2_{\hat{h}_n}/(g+1)} \exp\left[-\frac{|\hat{h}_n - \frac{ga+1}{g+1}u_n|^2}{\sigma^2_{\hat{h}_n}/(g+1)}\right] d\hat{h}_n, \\ &= \frac{1}{g+1}\exp\left[-\frac{g(a-1)^2}{(g+1)\sigma^2_{\hat{h}_n}}|u_n|^2\right].\end{aligned} \tag{27}$$

Replacing $\lambda_n(\phi)$ in (24) with (27), we have the symbol error probability for diversity receivers in estimated Ricean fading channels

$$\begin{aligned}P(E) = e^{-N\frac{K}{\rho^2}} \int_0^{\pi-\frac{\pi}{M}} \prod_{n=1}^{N} \left[1+\tilde{\gamma}_{rice}\frac{\sin^2(\frac{\pi}{M})}{\sin^2(\phi)}\right]^{-1} \times \\ \exp\left\{\frac{K}{\rho^2}\left[1+\tilde{\gamma}_{rice}\frac{\sin^2(\frac{\pi}{M})}{\sin^2(\phi)}\right]^{-1}\right\} d\phi\end{aligned} \tag{28}$$

where K is the Ricean factor, ρ is the covariance coefficient between the true CG and the estimated CG, and $\tilde{\gamma}_{rice}$ is defined in (26c). When $K = 0$, which corresponds to a Rayleigh fading channel, one can see that $\tilde{\gamma}_{rice} = \tilde{\gamma}_{ray}$ and (28) will specialize to the error probability for a Rayleigh fading channel given in (20).

V. Numerical Examples

The first example is used to validate the analytical error probability expressions derived for a system with a practical channel estimation algorithm. The channel estimation algorithm used in this example is the pilot assisted polynomial interpolation method with off-line training of [13], and the results are shown in Fig. 2 for 8PSK systems. One observes excellent agreement between the results obtained from Monte-Carlo simulation and analysis for various values of the Ricean factor K and the diversity order N.

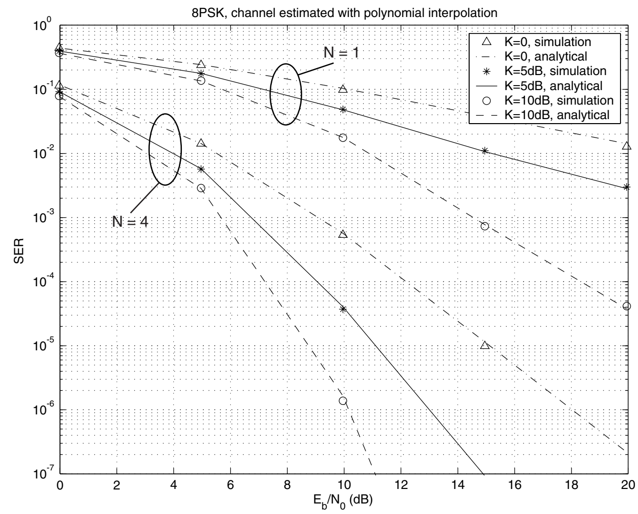

Fig. 2. The SER of 8PSK systems with polynomial interpolation channel estimation.

Next we evaluate the influence of channel estimation error on system performances. We are using the signal-to-channel estimation error ratio (SCER) η as the measure of the quality of the estimated channel since it is independent of the Ricean factor K for a certain channel estimation algorithm; it is defined as

$$\eta = \frac{E_s\Omega}{\sigma_e^2}. \tag{29}$$

The system error probabilities for different values of SCER are shown in Fig. 3 for Ricean fading channels. From the figure, one can see that the symbol error rates of all the systems decrease monotonically with the increase of SCER, as expected, but at different rates for different values of constellation size M and diversity order N. Observe from this figure that systems with higher diversity order and larger constellation size are more sensitive to channel estimation error, as expected. Therefore, more accurate channel estimation algorithms should be employed for systems with larger M or N.

The last example is used to study the relationship between channel estimation error and constellation size. Fig. 4 shows the SERs of systems with different constellation sizes versus the corresponding diversity orders. The absolute values of the curves' slopes are proportional to the value of SCER, and inversely proportional to the constellation size M. An interesting observation from Fig. 4 is that the SER performance of the system with SCER=$+\infty$ dB, $M = 8$ is close to the performance of the system with SCER$= 10$ dB and $M = 4$. The same observation holds for the curve with SCER$= +\infty$ dB, $M = 16$ relative to the curve with SCER$= 10$ dB and $M = 8$. This observation highlights the importance of having good channel estimation for MPSK systems operating in fading environments. Fig. 3 shows that SCER$= 25$ dB gives essentially the

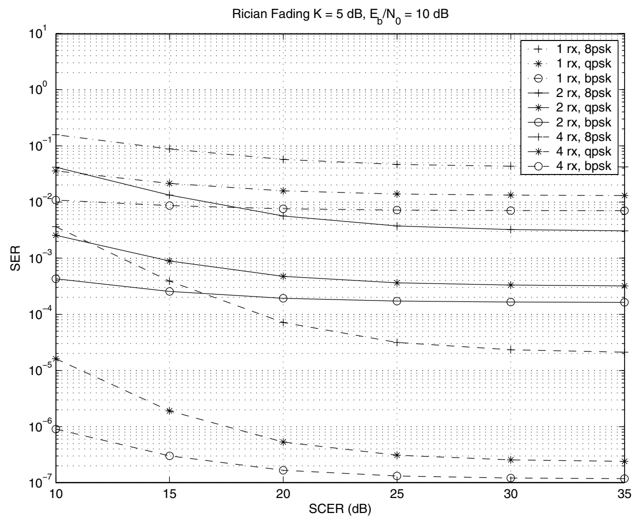

Fig. 3. The effect of SCER on system performance for Ricean fading channels.

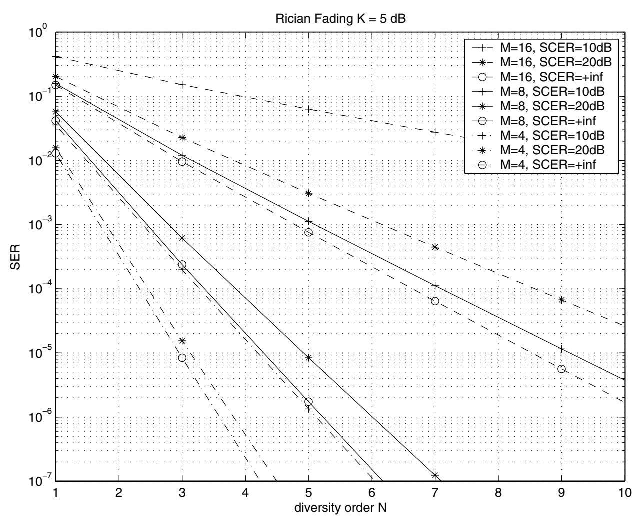

Fig. 4. The system performance for different constellation sizes and diversity orders.

same SER performance as SCER$= +\infty$ dB. Thus increasing the SCER from 10 dB to 25 dB allows doubling M while maintaining approximately the same SER.

VI. Conclusion

It has has been shown that the conventional MRC diversity receiver structure is not optimal when the channel estimation is corrupted by additive noise. A novel diversity receiver structure which is optimal for noisy channel state information has been derived. Exact, closed-form expressions for the average error probability of the optimal diversity receiver operating with noisy channel state information have been derived for MPSK modulation in both Rayleigh and Ricean channels. The new results for systems with noisy channel state information include systems with perfect channel state information as special cases. Simulation results are in excellent agreement with the theoretical results. A useful observation of significant practical design value was that improving the channel estimation SNR beyond 25 dB does not achieve worthwhile decrease in the SER. A second, interesting and useful observation was that improving the channel estimation SNR about 15 dB, from 10 dB to 25 dB allows doubling the constellation size while maintaining approximately the same SER.

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