Project 1 Simulation of Rayleigh fading

I. Objectives

1. Use the sum-of-sinusoid method to generate flat Rayleigh fading.
2. Investigate the statistical properties of the fading generated by simulator.

II. Theories

Let $h(t) = h_I(t) + j \cdot h_Q(t)$ denote the impulse response of flat Rayleigh fading. Both $h_I(t)$ and $h_Q(t)$ are zero-mean Gaussian distributed. Thus the fading envelope $|h(t)| = \sqrt{|h_I(t)|^2 + |h_Q(t)|^2}$ is Rayleigh distributed with pdf given by

$$f_{|h(t)|}(z) = \frac{2z}{s^2}\exp\left(-\frac{z^2}{s^2}\right)$$

where $s^2 = E\left(|h(t)|^2\right)$.

The time-domain property of the random process of Rayleigh fading can be characterized by its autocorrelation function. The autocorrelation function of Rayleigh fading satisfies $R_h(t) = E\left[h(t+t)h^*(t)\right] = J_0(2pf_D t)$, where $J_0(x)$ is the zero-order Bessel function of the first kind, and f_D is the maximum Doppler spread.

The random process of flat Rayleigh fading can be simulated with the sum-of-sinusoid method described as
follows:

$$h_I(nT_s) = \frac{1}{\sqrt{M}}\sum_{m=1}^{M}\cos\left\{2pf_D\cos\left[\frac{(2m-1)p+q}{4M}\right]\cdot nT_s + a_m\right\} \qquad (1)$$

$$h_Q(nT_s) = \frac{1}{\sqrt{M}}\sum_{m=1}^{M}\sin\left\{2pf_D\cos\left[\frac{(2m-1)p+q}{4M}\right]\cdot nT_s + b_m\right\} \qquad (2)$$

$$h(nT_s) = h_I(nT_s) + j\cdot h_Q(nT_s) \qquad (3)$$

where $q \in U(0,2p)$, $a_m \in U(0,2p)$, $b_m \in U(0,2p)$ are random variables uniformly distributed in $[0,2p]$, f_D is the maximum Doppler spread, T_s is sample period, and n is the sample index. (We cannot generate continuous-time signal with computer, so we need to use samples to approximate continuous-time signal.)

With the sum-of-sinusoid method described in (1)-(3), both the inphase components and quadrature components are zero-mean Gaussian distributed with variance 0.5.

III. Procedures

A. Fading simulator

1. For the sum-of-sinusoid method, $E\left[|h_I(t)|^2\right] = E\left[|h_Q(t)|^2\right] = 0.5$. Moreover, $h_I(t)$ and $h_Q(t)$ are independent. What is the value of $E\left[|h(t)|^2\right]$?

2. Based on the sum-of-sinusoid method described in (1)-(3), write a Matlab function to simulate flat Rayleigh fading. The function should have three input arguments:
   N: the number of samples to be generated.
   f_D: maximum Doppler spread.
   T_s: sample period.
   The output of the function should be the complex valued fading process. In the simulator development, using M = 16.

   Hint: to improve simulation speed, use as few loops as possible. Multi-level loops are strongly discouraged (Matlab is very slow in processing loops). Try to use vectors and matrices for operations over a series of numbers or variables.

   Example 1:
   ```
   x = 0;
   for m = 1:10
     x = x+m;
   end
   ```
   can be replaced by
   ```
   a = 1:10;
   x = sum(x);
   ```

   Example 2: a = [1:10], b = [11:20];
   ```
   for m = 1:length(a)
     x(m) = a(m)*b(m);
   end
   ```
   can be replaced by
   ```
   x = a.*b;
   ```

   Example 3: a = [1:10], b = [11:20];
   ```
   x = 0;
   for m = 1:length(a)
     x =x+ a(m)*b(m);
   end
   ```
   can be replaced by
   ```
   x = a*b';
   ```

3. Using the Rayleigh fading simulator generate N = 400 samples, set Ts = 0.0001 seconds, f_D = 100 Hz.
   h = Rayleigh(N, fd, Ts)
   Plot the real part (real(h)), imaginary part (imag(h)), amplitude (abs(h)), phase (angle(h)) of the flat fading as a function of time (the x-axis is [0:N-1]*Ts).

4. Repeat step 3 by setting f_D = 10 Hz.

5. Repeat step 3 by setting f_D = 300 Hz.

B. Statistical Properties

1. Using the Rayleigh fading simulator generate N = 100,000 samples, set T_s = 0.0001 seconds, f_D = 100 Hz. Using the function mean( ) and var( ) evaluate the mean, variance of the real part and imaginary part of the fading samples. How do they compare with their theoretical values?

2. Using the function pdf evaluate the pdfs of the real part, imaginary part of the Rayleigh fading. Plot the pdfs in two different figures. Plot the theoretical Gaussian pdfs in the corresponding figures.

3. Using the function pdf evaluate the pdf of the amplitude of the Rayleigh fading samples. Plot the pdf. Plot the theoretical Rayleigh pdf in the same figure. What value of $s^2 = E\left(|h(t)|^2\right)$ should be used in the theoretical calculation?

4. In this step, we are going to evaluate the auto-correlation function.
   a) Generate a flat Rayleigh fading process with parameters N = 1,000, Fd = 180Hz, Ts = 0.0001. Assign the process to variable *h*.
   b) Evaluate the time domain auto-correlation function with the Matlab function *corr_mat(1, :) = xcorr(h)*. The correlation information is stored in the first row of the matrix *corr_mat*.
   c) Use for loop, repeat the above two steps 100 times to generate 100 Rayleigh fading processes. For the kth loop, the correlation information is stored in the kth row of corr_mat.
   d) Evaluate the ensemble average of the auto-correlation function by using the command mean(corr_mat, 1). The second parameter ‘1’ tells Matlab that the mean operation is performed column wise of the matrix.
   e) Plot the ensemble average of the auto-correlation function.
   f) *Bonus question*: plot the theoretical value of the autocorrelation function in the same figure. The Matlab function besselj( ) corresponds to the Bessel function.