Integrated Transceivers for Error Reduction in MIMO Wireless Communications

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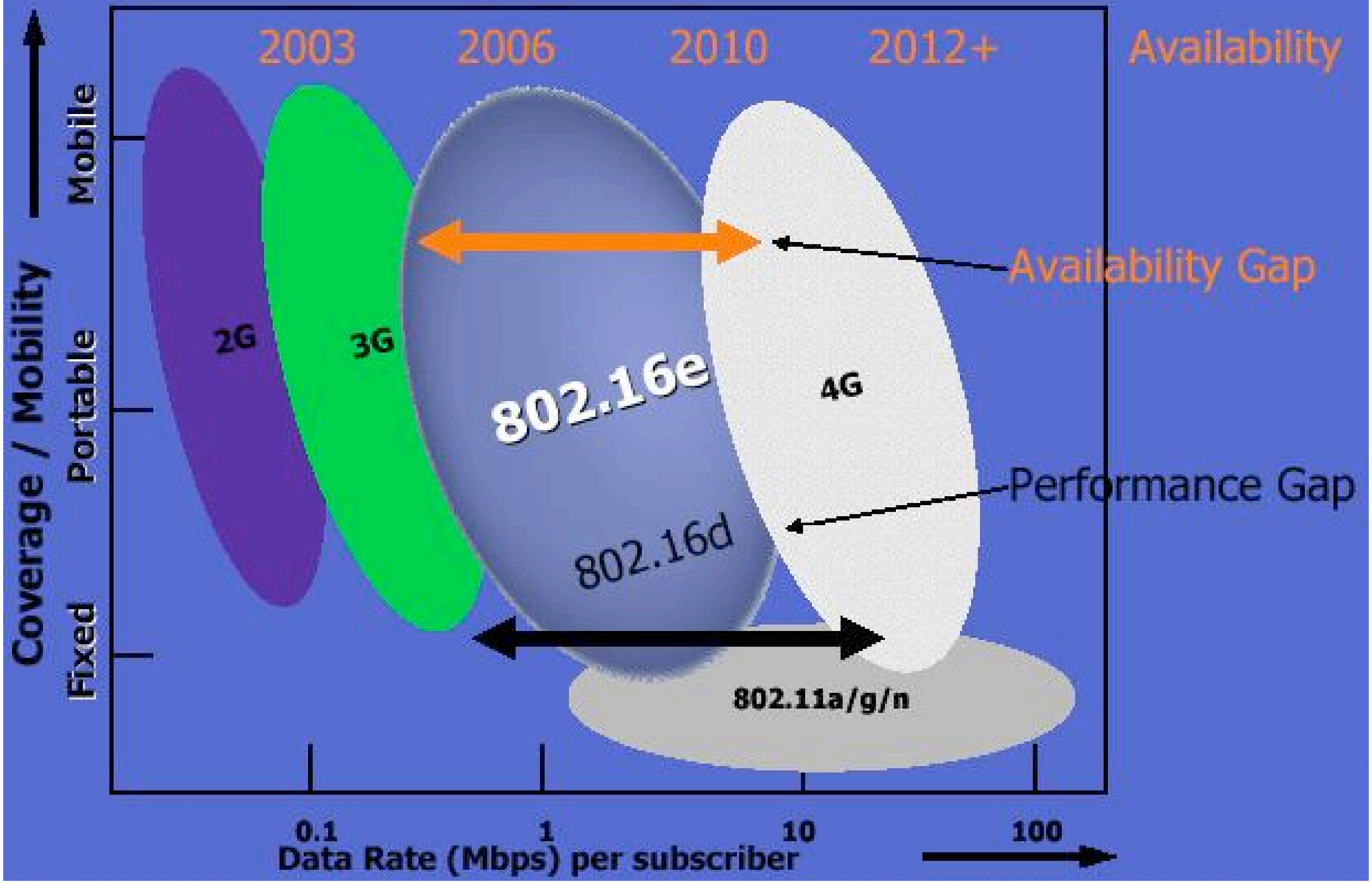
Explosive Wireless Communications Industry

- Rapid expansion of cellular users with over $1 trillion/year market.
- One of the fastest growing commercial sectors in the past 15 years.
- The number of wireless subscribers has *exceeded* wireline.
- WIFI has also taken WLAN to a new level.
- Future WLAN (e.g., WiMax) technologies will continue the rapid wireless advances.
- Future Advanced Wireless Systems must deliver
  - *Better spectral efficiency;*
  - *Performance and quality guarantee;*
  - *Power and battery conservation;*
  - *Interference minimization;*
  - *Multimedia Service.*

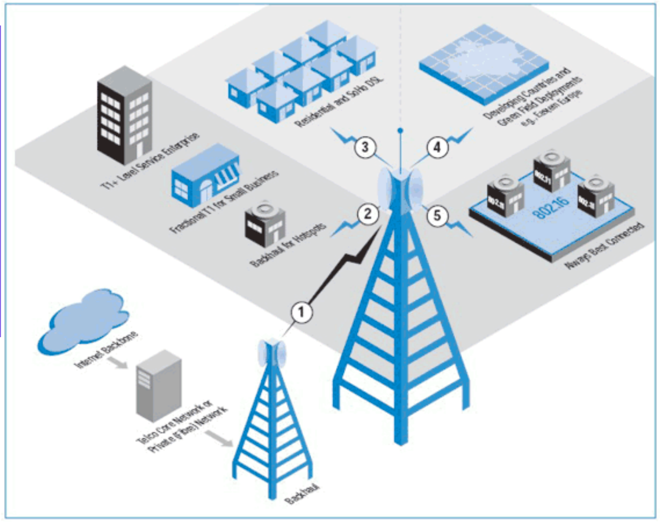
Broadband Wireless Market Opportunity



Wi-Fi (802.11n)

- 802.11 (1, 2Mb/s) -> 802.11b (up to 11 Mb/s) -> 802.11a,g (up to 54 Mb/s) -> 802.11n (65 Mb/s per stream, 300 Mb/s with 2x2 MIMO)
- 1st large scale deployment of MIMO technology
- IEEE 802.11n standard draft 2.0 approved by WG.
- Over 95 products are Wi-Fi CERTIFIED (draft 2.0)
- Up to 600Mb/s (theoretical) rate with 4x4 MIMO configurations

WiMAX (IEEE802.16)

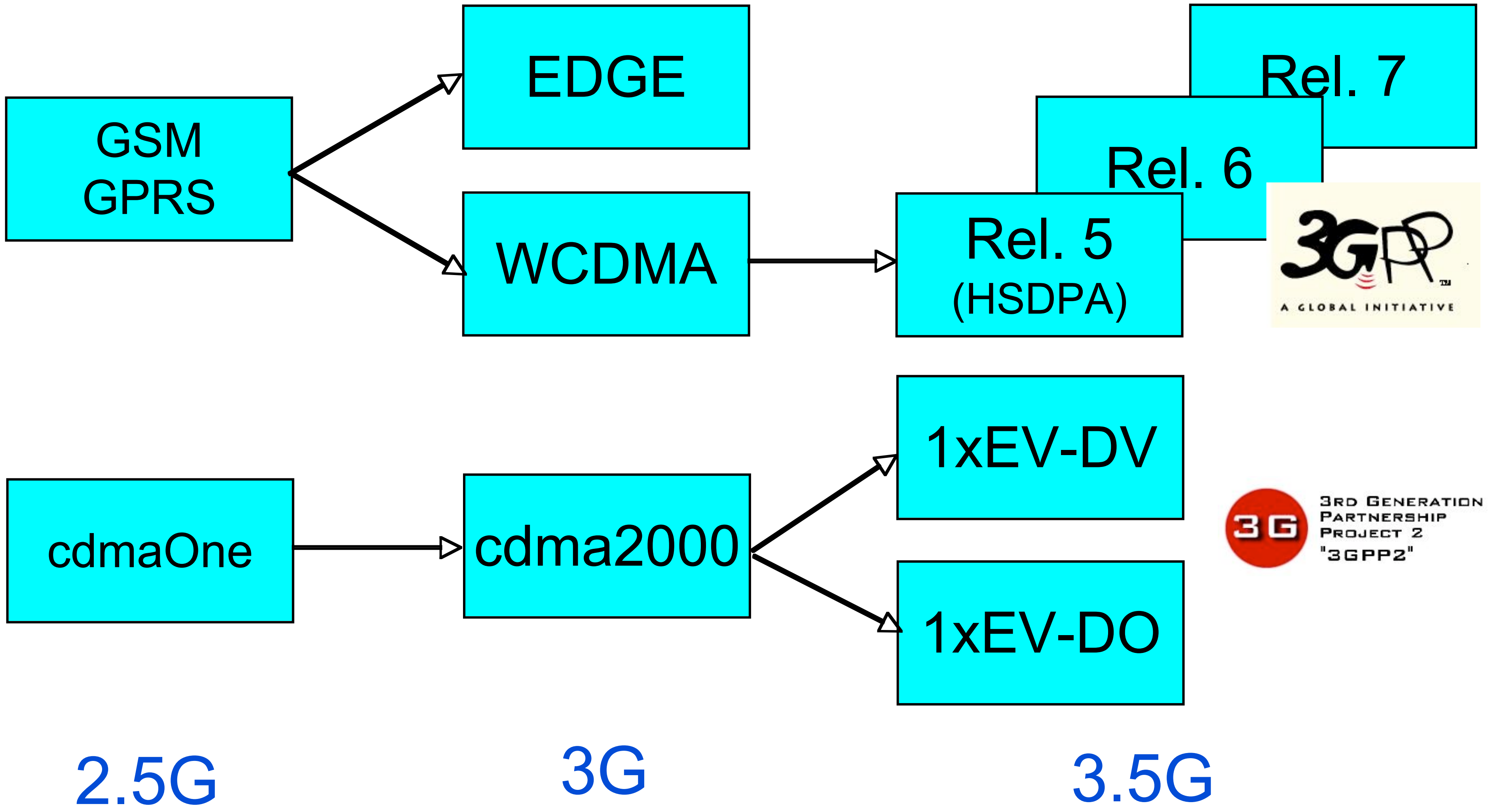


- Worldwide Interoperability for Microwave Access: Goal to provide long distance wireless data services

- Broad applications of MIMO technologies
- IEEE 802.16-2004 standard for fixed wireless (<63Mb/s downlink and 28Mb/s uplink in 10MHz bandwidth), end-to-end IP based QoS.
- IEEE 802.16e-2005 standard for mobile wireless apps.

World-wide Deployment of WiMAX

Evolution to 3G and B3G



3G LTE (Long Term Evolution)

- The Internet is the driving force for higher data rates and high speed access for mobile wireless users.
- The market may not support two wireless mobile technologies such as 4G wireless cellular networks and mobile WiMAX.
- OFDM and MIMO are the largest strongest candidates for 4G access technologies.
- NTT DoCoMo and Samsung are rolling out so called “3.99G” technologies.   Lessons for 4G in the next 4 years.
- The lack of radio spectrum suitable for 4G deployments will be a major impediment to the migration of 3G to 4G networks.
- Hence, Long Term Evolution of the 3GPP cellular networks (up to 100Mb/s on UL and 50Mb/s on DL)

Error Protection Mechanisms in Wireless Systems

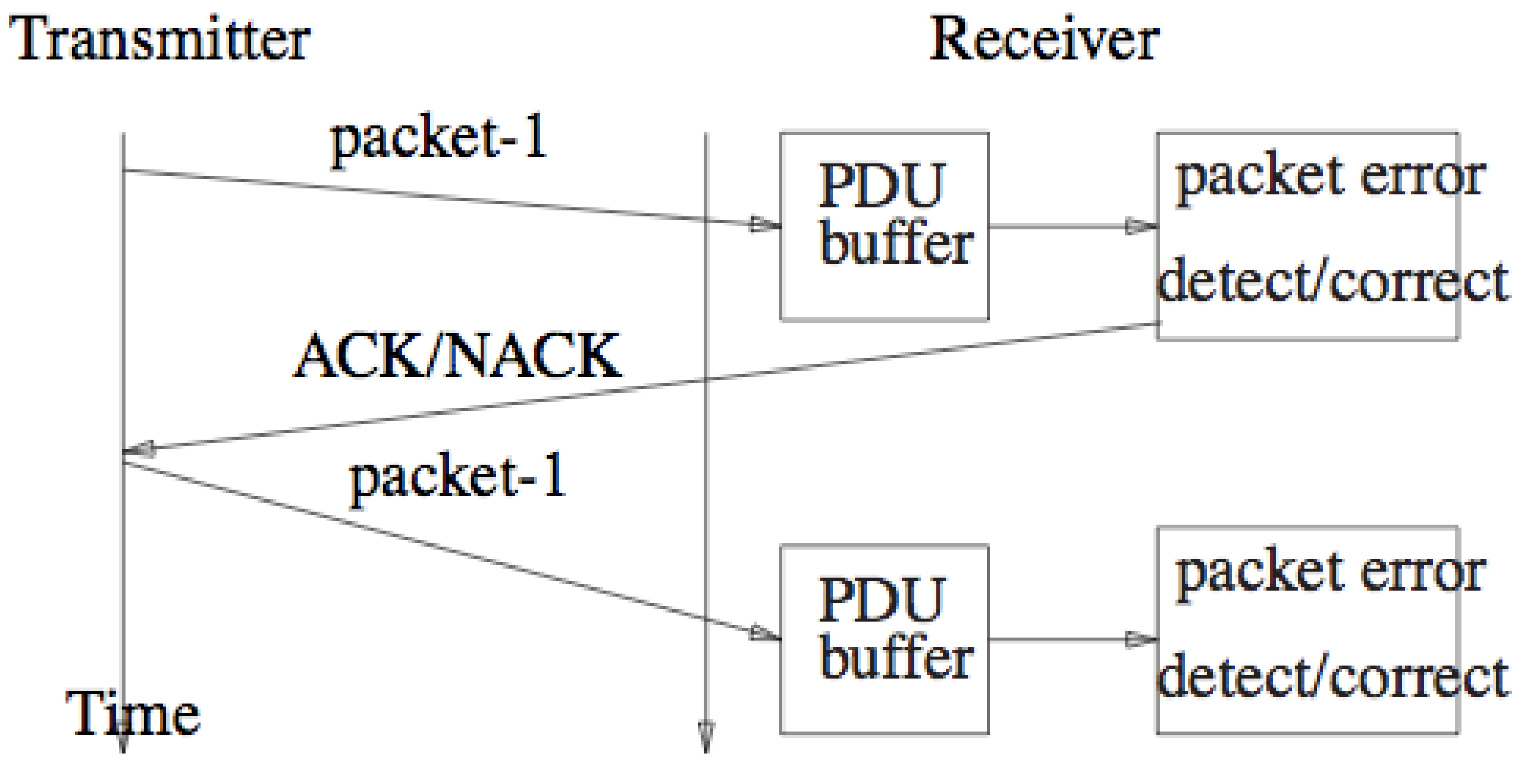
- FEC
  - Transmission of redundant symbols to be used for error correction
  - Typically coding rate is fixed, leading to predetermined redundancy rate regardless of actual number of bit errors.
  - Highly powerful tool that can be jointly operated with transceiver design (trellis codes, turbo equalization).
- ARQ
  - Incremental redundancy ONLY when “NACK” is received
  - Feedback channel must exist
  - Rate is adaptive
  - Efficiency-delay tradeoff
  - Traditionally not integrated with transceiver operation

Hybrid ARQ inclusions

- 802.11n has included hybrid ARQ for packet error correction
- WiMAX has hybrid ARQ provisions
- 3GPP LTE also adopted hybrid ARQ for packet error correction.
- Better analysis and designs of Hybrid ARQ mechanisms NEEDED.

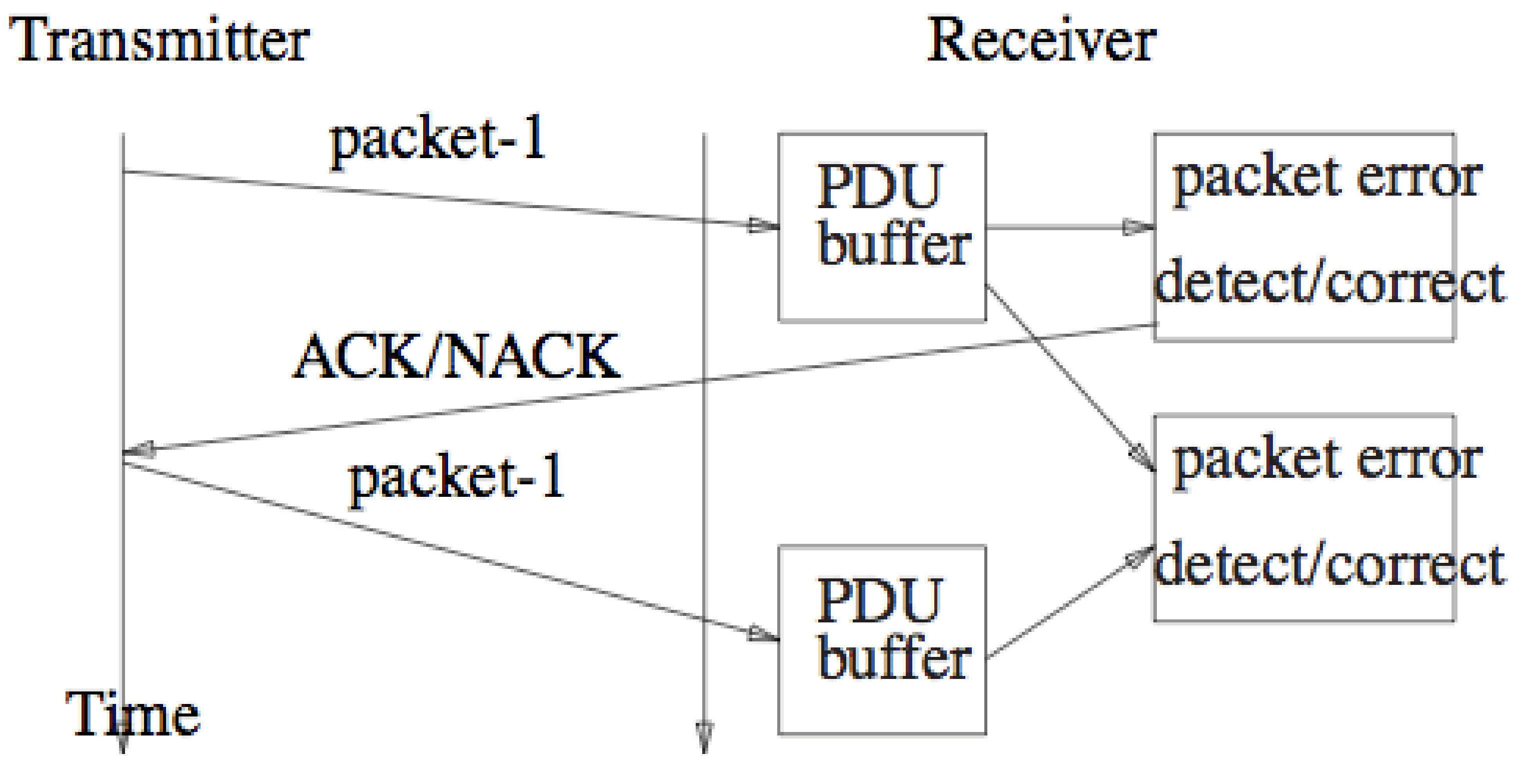
Type-I Hybrid ARQ (Simple Repeat)

- Receiver discards erroneous data



Type-II Hybrid ARQ

- Receiver buffer all previous transmissions
- Joint decoding of all (partial) transmissions



New Design Concepts in Hybrid ARQ

- Motivation:
  - Most ARQ protocols resemble a form of delayed coding, restrictively view retransmissions as additional parity information, and
  - Signal processing is replete with diversity methods

- Apply advanced transceiver techniques towards
  - pre-processing of retransmission at the transmitter
  - combining receivers that exploit the pre-processing and the independent channel and noise realizations experienced by retransmissions
  - designing and integrating effective ARQ protocols for MIMO wireless.

NEW Hybrid ARQ + MIMO Integration

- Symbol Mapping Diversity and Joint Detection
- Joint Hybrid ARQ Transmitter Precoding
- Joint hybrid ARQ-MIMO receivers.
- Bandwidth Efficient HARQ with Embedded Space-time Codes

ARQ Mapping Diversity for MIMO Wireless

- MIMO has been viewed by many as a future direction of wireless communications.
- Significant increase of channel capacity has been shown.
- Major progresses in MIMO wireless include D-BLAST, V-BLAST, and Space-Time coding.
- WiMAX and many future services will incorporate MIMO technologies
- New MIMO WLAN 802.11n is already in the market.

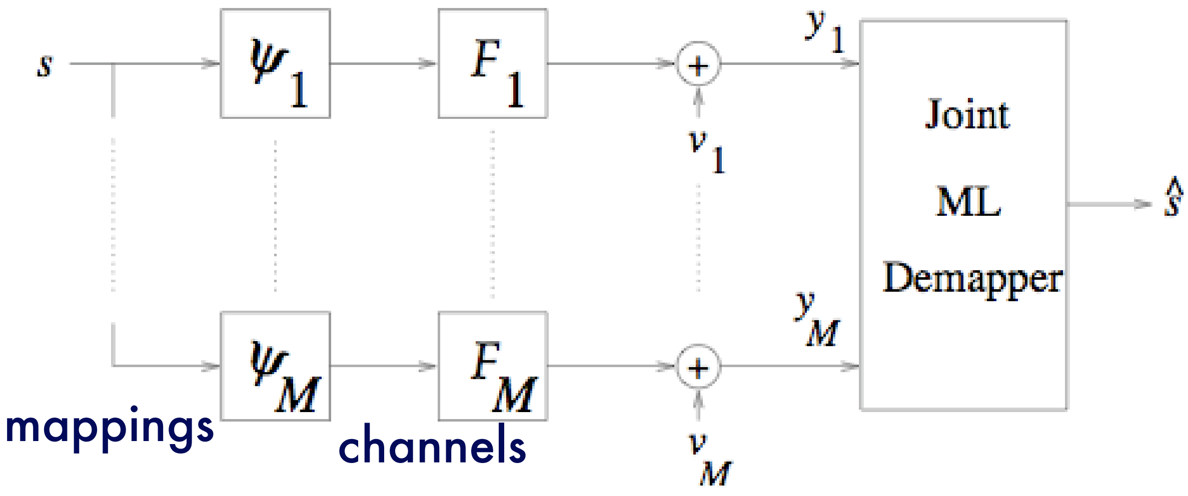
- *Efficient HARQ-MIMO integration takes advantages of new diversity with MIMO*

Symbol Mapping Diversity in ARQ

- Traditional ARQ repeats the previous packet.
- Mapping diversity + joint ARQ detection [Samra, Ding, Hahn] demonstrates simplicity and gain.
- Each packet retransmission uses a different bit-to-symbol mapping

Label s is transmitted M times; transmitter sends symbols $\psi_1[s], \ldots, \psi_M[s]$.

Receiver obtains samples $y_m = F_m(\psi_m[s]) + v_m$, where $m = 1, \ldots, M$ and v_m is $\mathcal{CN}(0, \sigma_v^2)$. Assume that $v_1, \ldots, v_M$ are independent.



Symbol Mapping Optimization

- Find optimum mapping for each packet (re)transmission by minimizing the BER (bound) using QAM/PSK;
- If a bit stream s is transmitted M times, find the mappings

$$\psi_1(s),\ \psi_2(s),\ \cdots,\ \psi_M(s)$$

- Assuming joint Maximum Likelihood detection, optimization can be carried out for different fading channels: Rayleigh, Ricean, flat fading, by minimizing union bound (PEP.).
- This optimization can be done a priori.

Insight on Mapping Diversity of ARQ

Effectiveness of mapping diversity is easily apparent from the PEP expressions it produces for AWGN channels

$$Q\left(\sqrt{\frac{1}{2\sigma_v^2}\sum_{m=1}^{M}|d_m[s,k]|^2}\right).$$

with $d_m[s,k] = \psi_m[s] - \psi_m[k]$, and for fading channels

$$E_{\mathbf{h}}\left\{Q\left(\sqrt{\frac{1}{2\sigma_v^2}\sum_{m=1}^{M}|h_m|^2|d_m[s,k]|^2}\right)\right\}.$$

High dependence upon the combined squared Euclidean distance (CSED) between the mapped symbols across the M transmissions.

Complexity

- Given L bits in each transmitted data label, 2^L possible mapping are available for each distinct label.
- Number of possible selections in each mapping $2^L!$
- Given M transmission, the total search space is

$$(2^L!)^M$$

- Fortunately in ARQ, only the current mapping is unknown
    - previous mapping has already been sent
    - future mapping may not take place
- Should only consider current mapping optimization
- Use quadratic assignment problem (QAP) to find good approximate solutions.

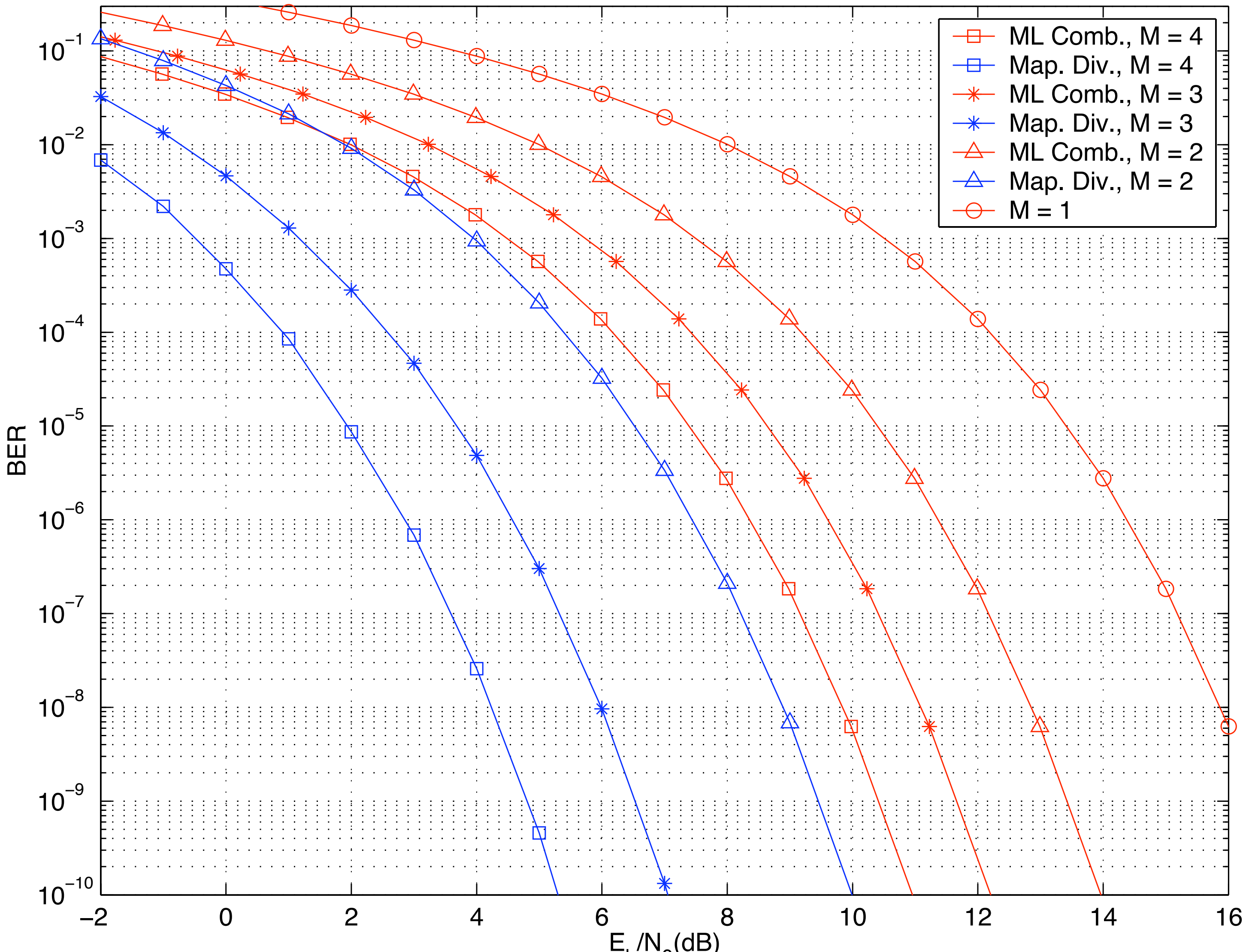
SISO Results

- 16-QAM, 10dB, AWGN

0000 1110 1000 1100	1000 1000 0011 0001	1001 1010 0010 0010	0001 1100 1101 0111
0100 0101 0110 1010	1100 0011 1001 1111	1101 0001 1100 0100	0101 0111 0101 1000
0110 0100 0111 1001	1110 0010 1110 0101	1111 0000 1011 1110	0111 0110 0100 1011
0010 1111 1111 0110	1010 1001 0000 0011	1011 1011 0001 0000	0011 1101 1010 1101

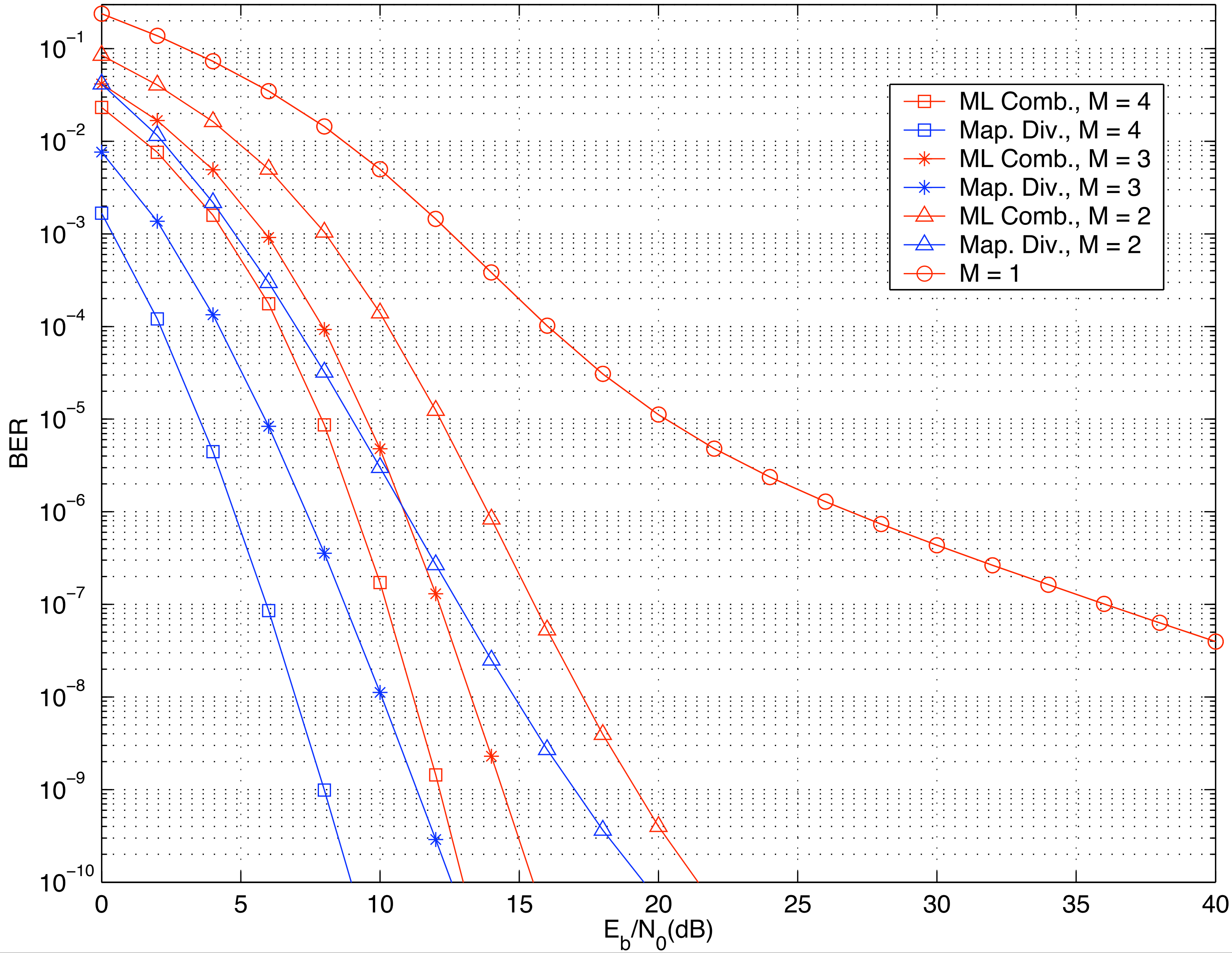
BER Results

- Significant BER improvement



BER Results

- Significant BER improvement

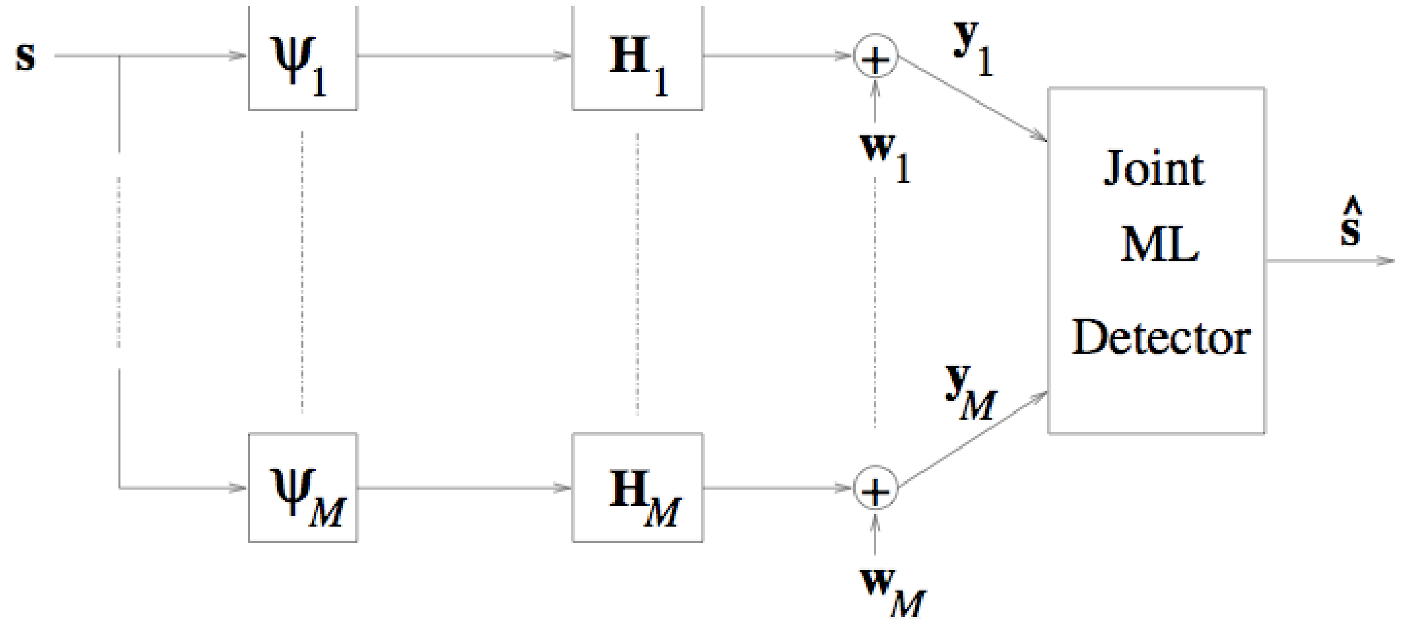


Mapping Diversity Gains

- Very simple to implement
  - Transmitter and receiver agree on mapping sequences
  - Apply table-look up during "NACK"
  - Joint detection requires minimum complexity increase
- Can be integrated with OFDM, MIMO, and any ARQ-ready systems.

MIMO-ARQ Mapping Diversity

- Each symbol vector **s** is sent M times.
- Optimize mapping diversity as in flat fading SISO channels
- Joint detection is performed at the receiver



Joint MIMO-ARQ Detection

- Receiver (over M transmissions) have

$$\mathbf{y}_m = \begin{bmatrix} y_{m,1} \\ \vdots \\ y_{m,K} \end{bmatrix} = \mathbf{H}_m \begin{bmatrix} \psi_m[s_1] \\ \vdots \\ \psi_m[s_N] \end{bmatrix} + \begin{bmatrix} w_{m,1} \\ \vdots \\ w_{m,K} \end{bmatrix} \quad (1)$$

$$= \mathbf{H_m}\vec{\psi}_m[\mathbf{s}] + \mathbf{w}_m.$$

Receiver can employ a joint ML decoding

$$\hat{\mathbf{s}} = \arg\min_{\mathbf{s}} \sum_{m=1}^{M} ||\mathbf{y}_m - \mathbf{H}_m\vec{\psi}_m[\mathbf{s}]||^2.$$

- Sphere-decoding to reduce complexity of ML.

Sphere Decoding in Joint MIMO-ARQ Detection

- Sphere decoding approximately reduces decoding complexity from L^N to N^3
- Define $\mathbf{p}_m = \mathbf{H}_m^\dagger \mathbf{y}_m$ and upper-triangular matrix $\mathbf{U}_m$ so that $\mathbf{U}_m^H \mathbf{U}_m = \mathbf{H}_m^H \mathbf{H}_m$. Metric for minimization becomes:

$$\sum_{n=1}^{N} \sum_{m=1}^{M} u_{m,nn}^2 \left| \psi_m[s_n] - p_{m,n} + \sum_{k=n+1}^{N} \frac{u_{m,nk}}{u_{m,nn}} (\psi_m[s_k] - p_{m,k}) \right|^2$$

Define a hypersphere of radius R centered by $\mathbf{p}_1, \ldots, \mathbf{p}_M$.

Iteratively select estimates $\hat{s}_N, \ldots, \hat{s}_1$ within hypersphere.

Select the estimate $\hat{s}_n$ from the set $\mathcal{S}_n$ of labels

that fall inside the hyper-ellipsoid region $\mathcal{E}_n$ defined by

$$\sum_{m=1}^{M} u_{m,nn}^2 \left| \psi_m[s_n] - a_m \right|^2 < r_n^2,$$

Sphere decoding (Continued)

Compute a_m and r_n from the existing estimates $\hat{s}_{n+1}, \ldots, \hat{s}_N$:

$$a_m = p_{m,n} - \sum_{k=n+1}^{N} \frac{u_{m,nk}}{u_{m,nn}} (\psi_m[\hat{s}_k] - p_{m,k})$$

$$b_m = \sum_{k=n+1}^{N} u_{m,kk}^2 \left| \psi_m[\hat{s}_k] - p_{m,k} + \sum_{t=k+1}^{N} \frac{u_{m,kt}}{u_{m,kk}} (\psi_m[\hat{s}_t] - p_{m,t}) \right|^2 ,$$

$$r_n^2 = R^2 - \sum_{m=1}^{M} b_m .$$

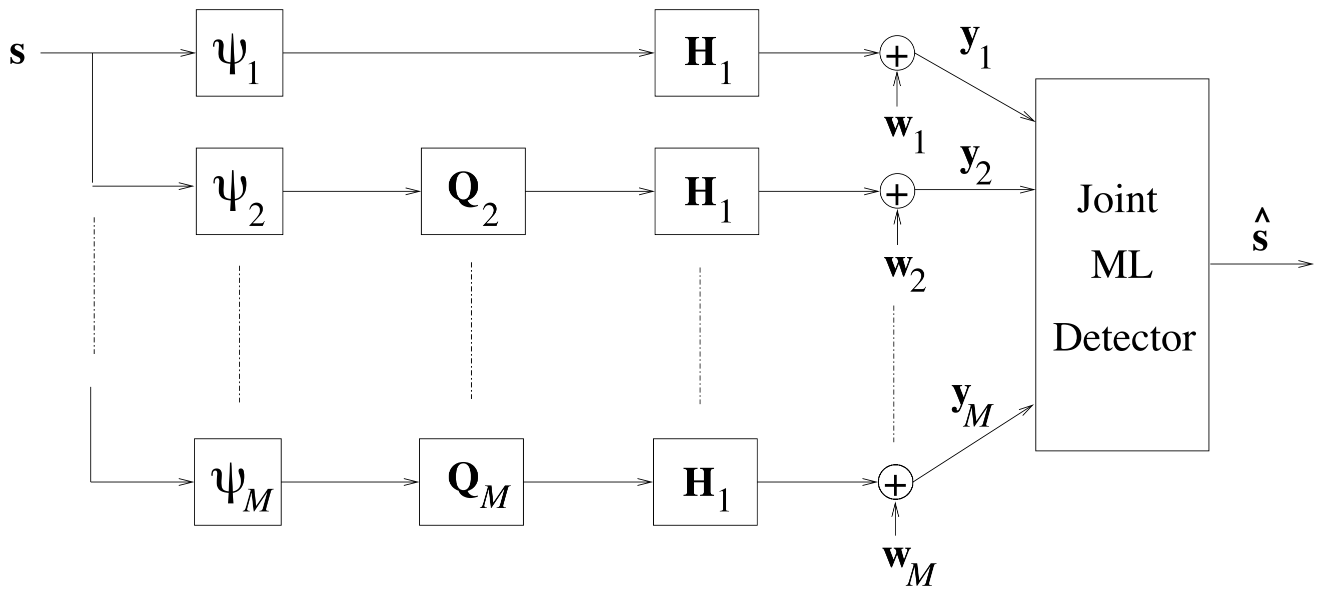
If no label $\hat{s}_n$ exists within the region, invalidate the estimate $\hat{s}_{n+1}$, Choose a new estimate from $\mathcal{S}_{n+1}$. If $\mathcal{S}_{n+1}$ is empty, we retreat to $\mathcal{S}_{n+2}$, etc.

When $\hat{s}_1$ is chosen, its distance becomes the new R, and process is repeated to find better estimates.

Key to fast performance is quick enumeration of candidates within $\mathcal{E}_n$.

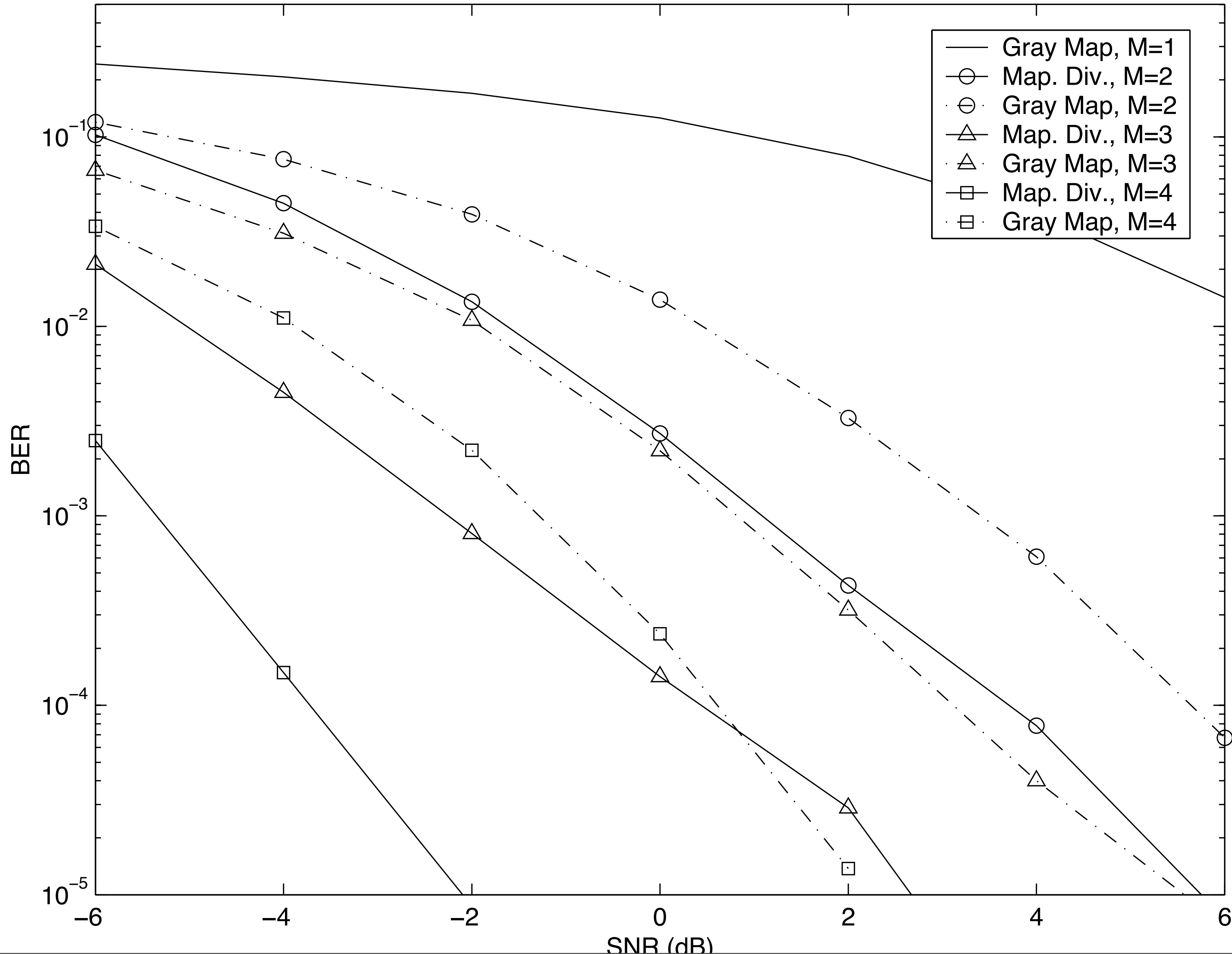
Discussions

- Good technique exists for M=1 [Hochwald, ten Brink]
- For ARQ with *M transmissions,* we define a new method using bounding box.
- Sphere decoding suggests that each label in the MIMO transmission can be treated separately, justifying mapping using flat fading SISO.
- For MIMO channels that are static, precoding before



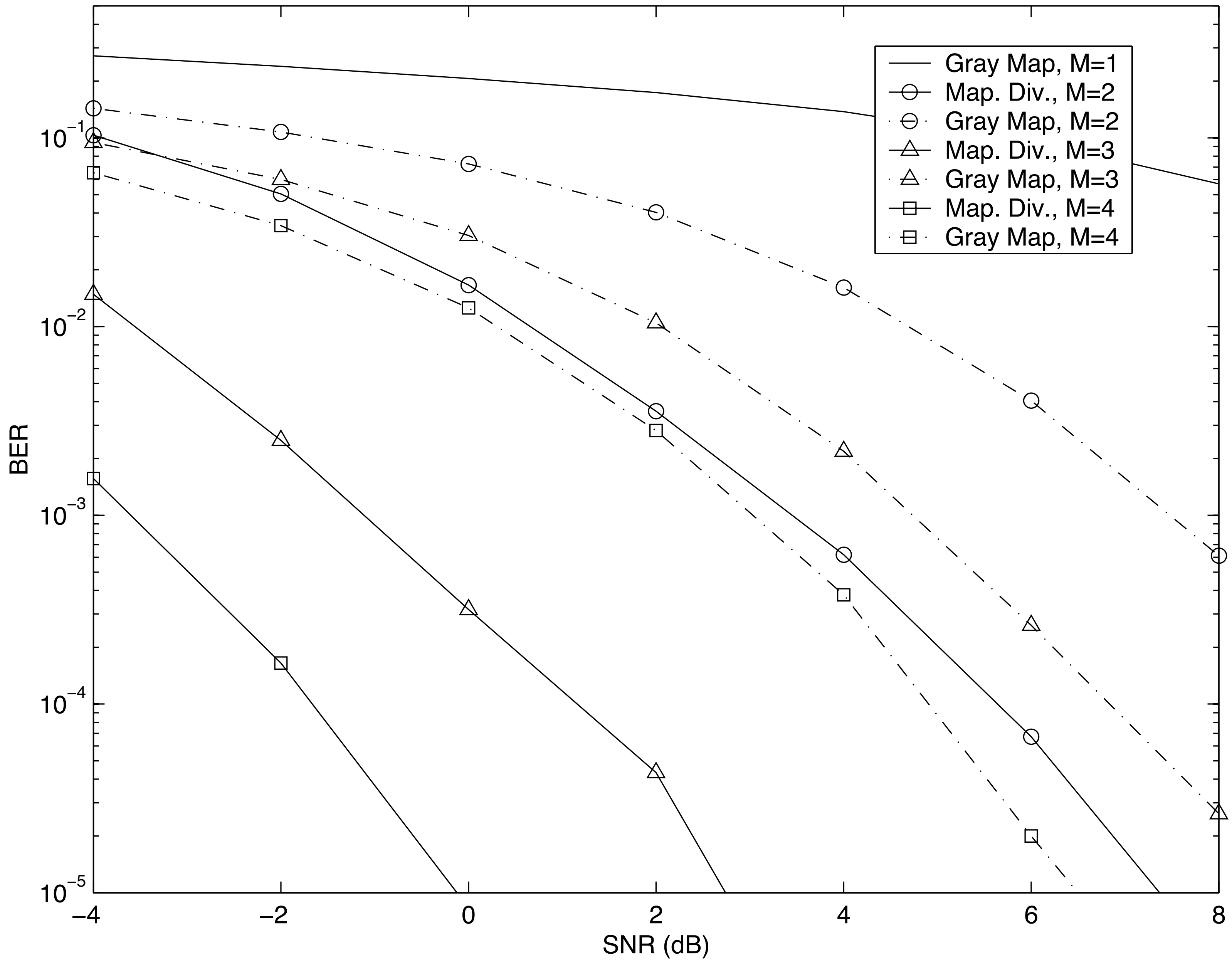
MIMO ARQ Simulation Results with Mapping Diversity

- 100000 symbol vectors, 4x4 dynamic channels, QAM-16



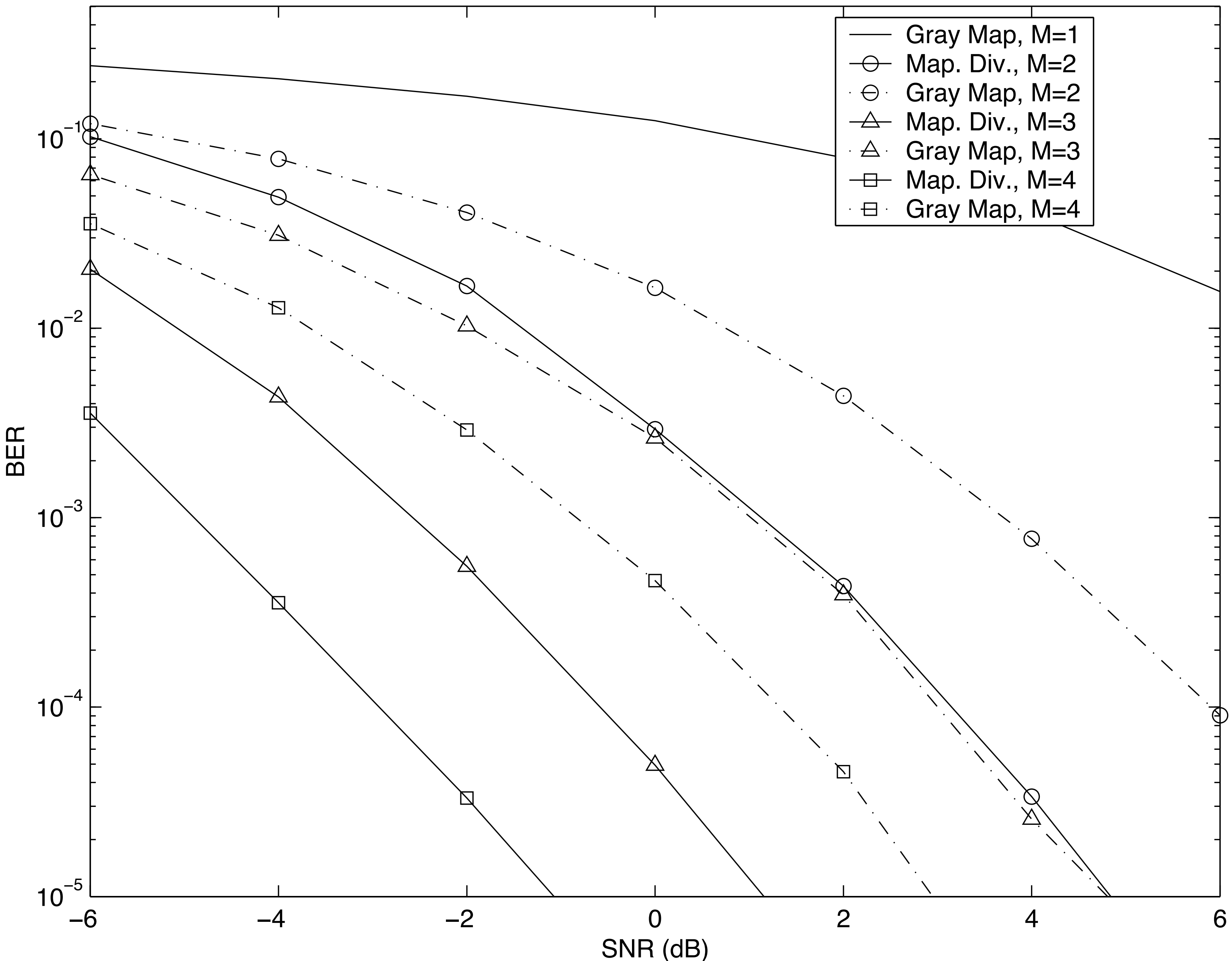
MIMO ARQ Simulation Results with Mapping Diversity (2)

- 100000 symbol vectors, 4x4 dynamic channels, QAM-64



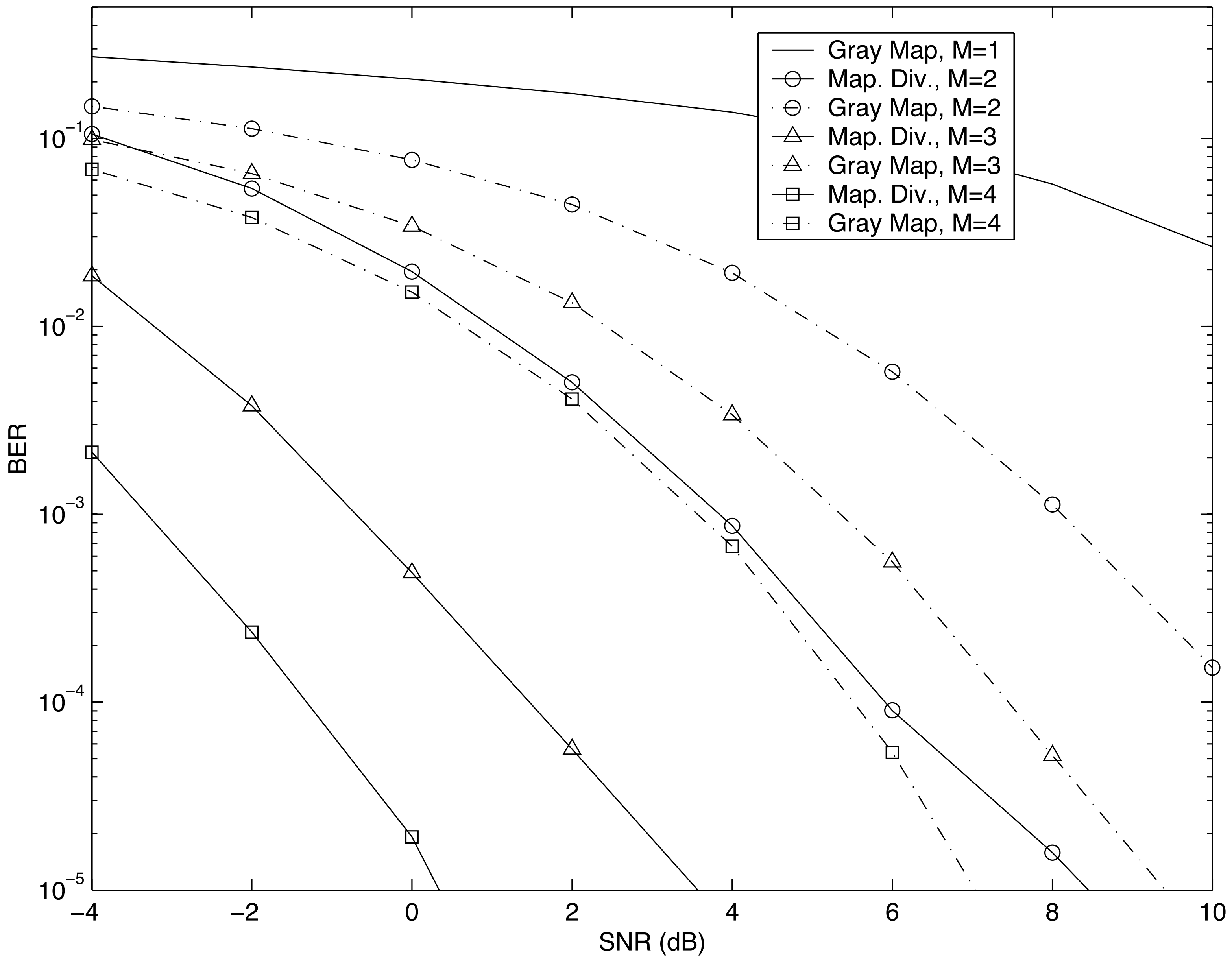
MIMO ARQ Simulation Results with Mapping Diversity (3)

- 100000 symbol vectors, 4x4 static channels, QAM-16



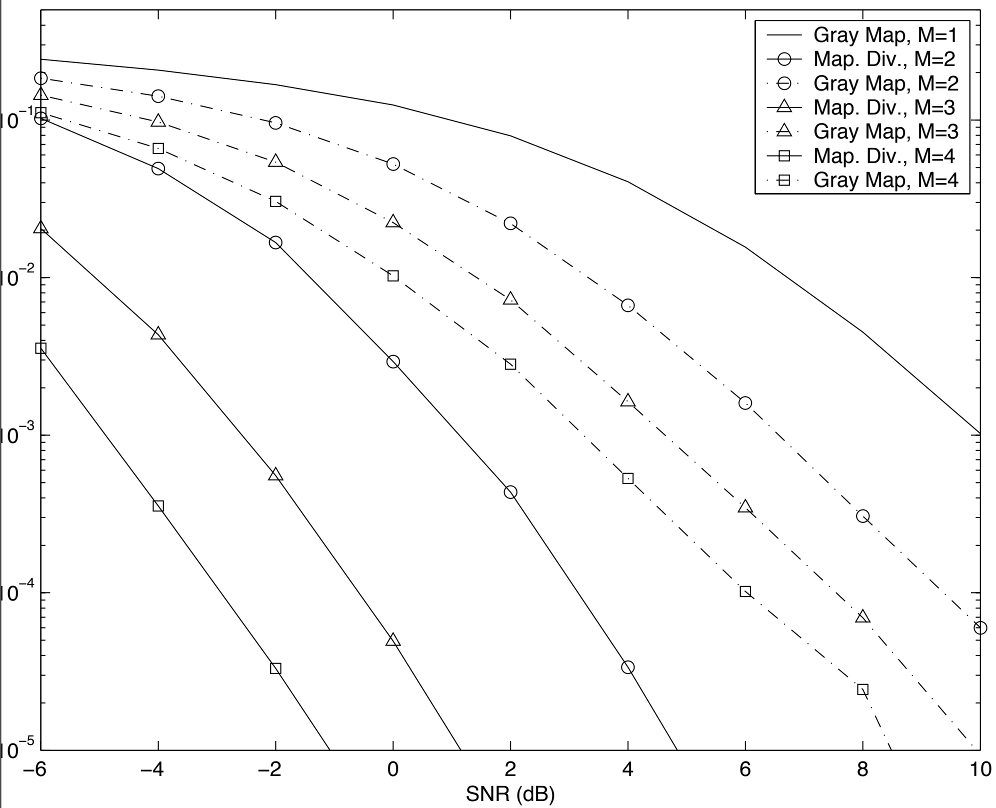
MIMO ARQ Simulation Results with Mapping Diversity (2)

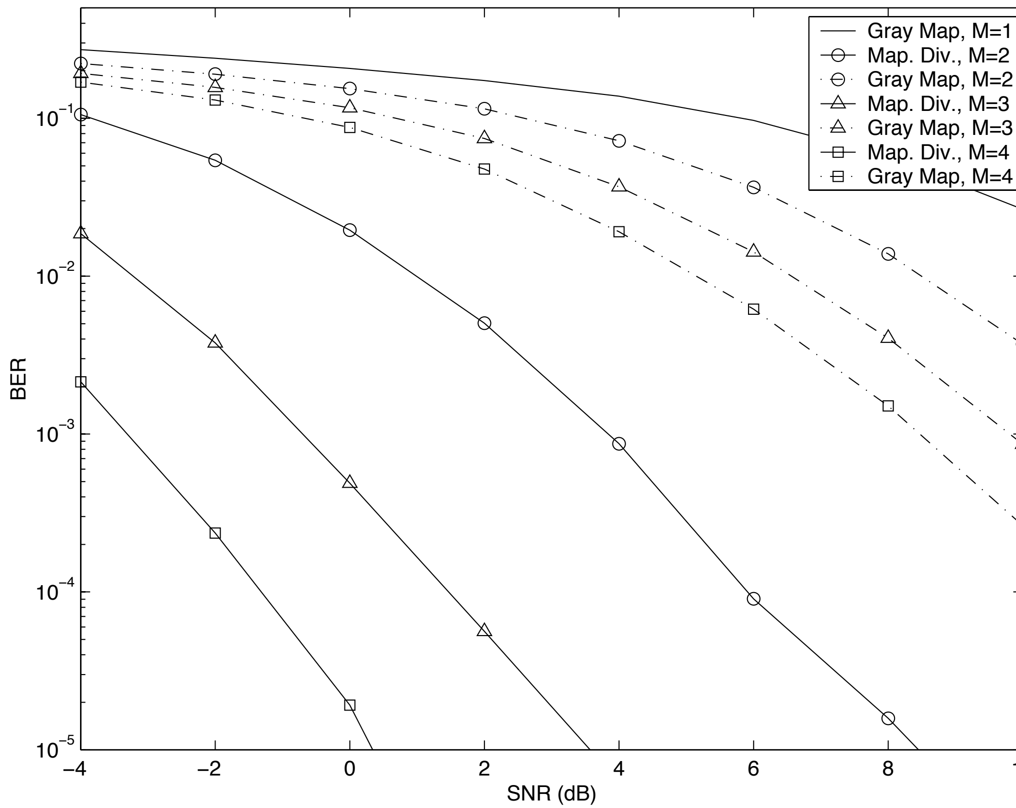
- 100000 symbol vectors, 4x4 dynamic channels, QAM-64



4x4 full rate Space-time code

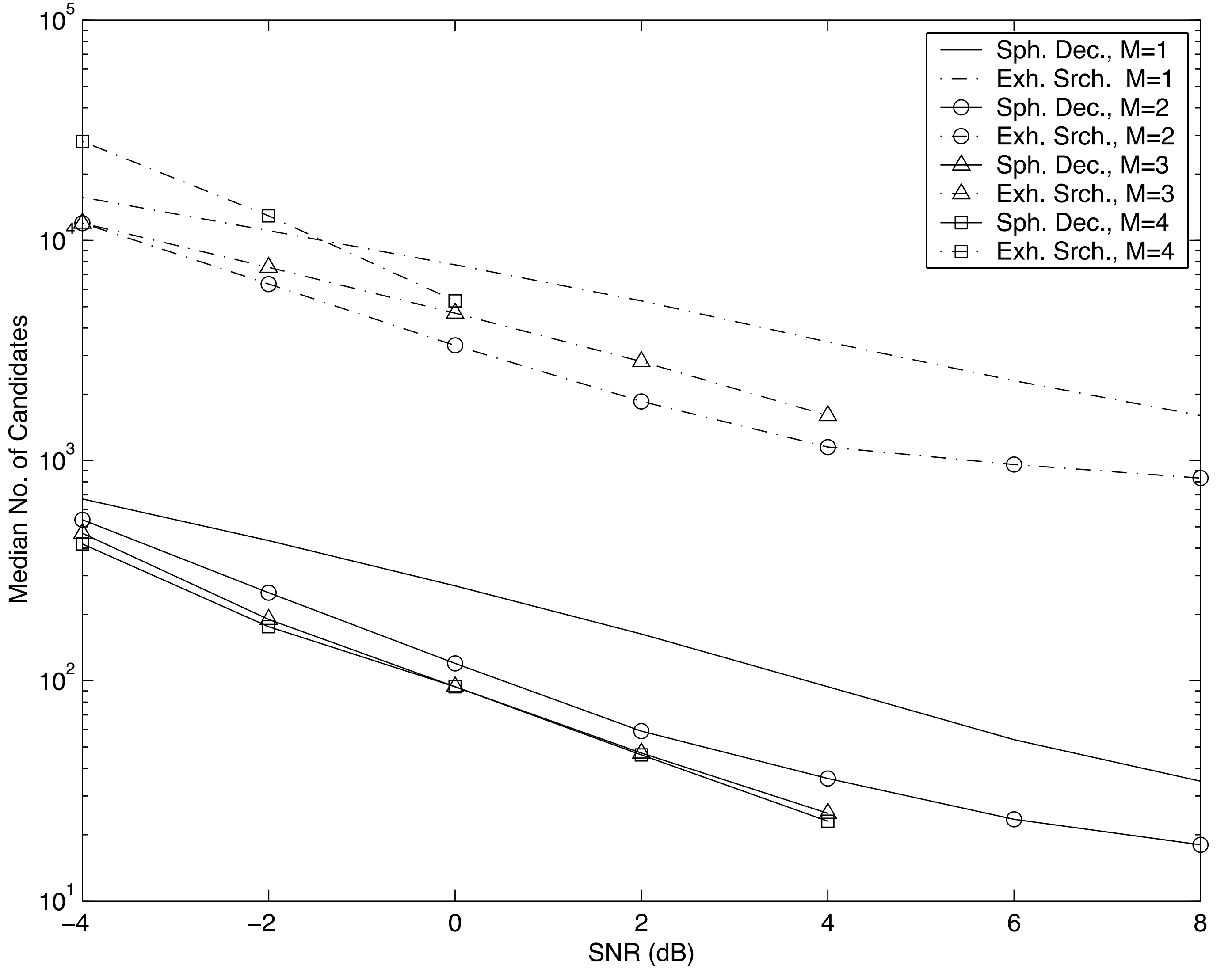
- 4x4 and full rate STBC + mapping diversity
- 16-QAM 64-QAM





Complexity Results

- Median number of candidates (64-QAM, 4x4 variable channels): comparing exhaustive search versus our new method

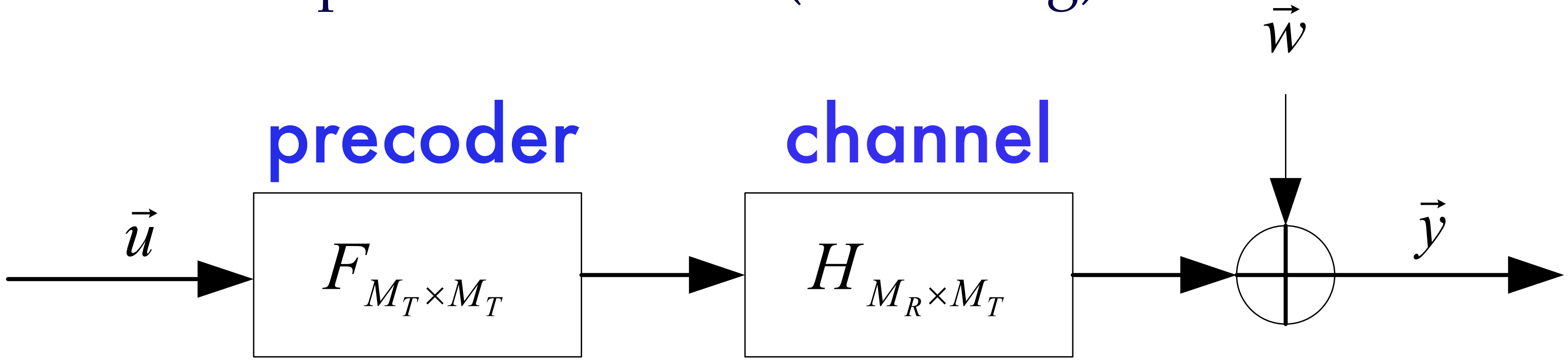


What about frequency-selective channels

- Apply OFDM (cyclic prefix) to yield multiple flat fading channels;
- Utilize mapping diversity carrier-by-carrier;
- Mapping diversity can also be applied with integrated (turbo equalizer)
- OFDM applies FFT as linear precoding; general linear precoding + mapping diversity can be developed.

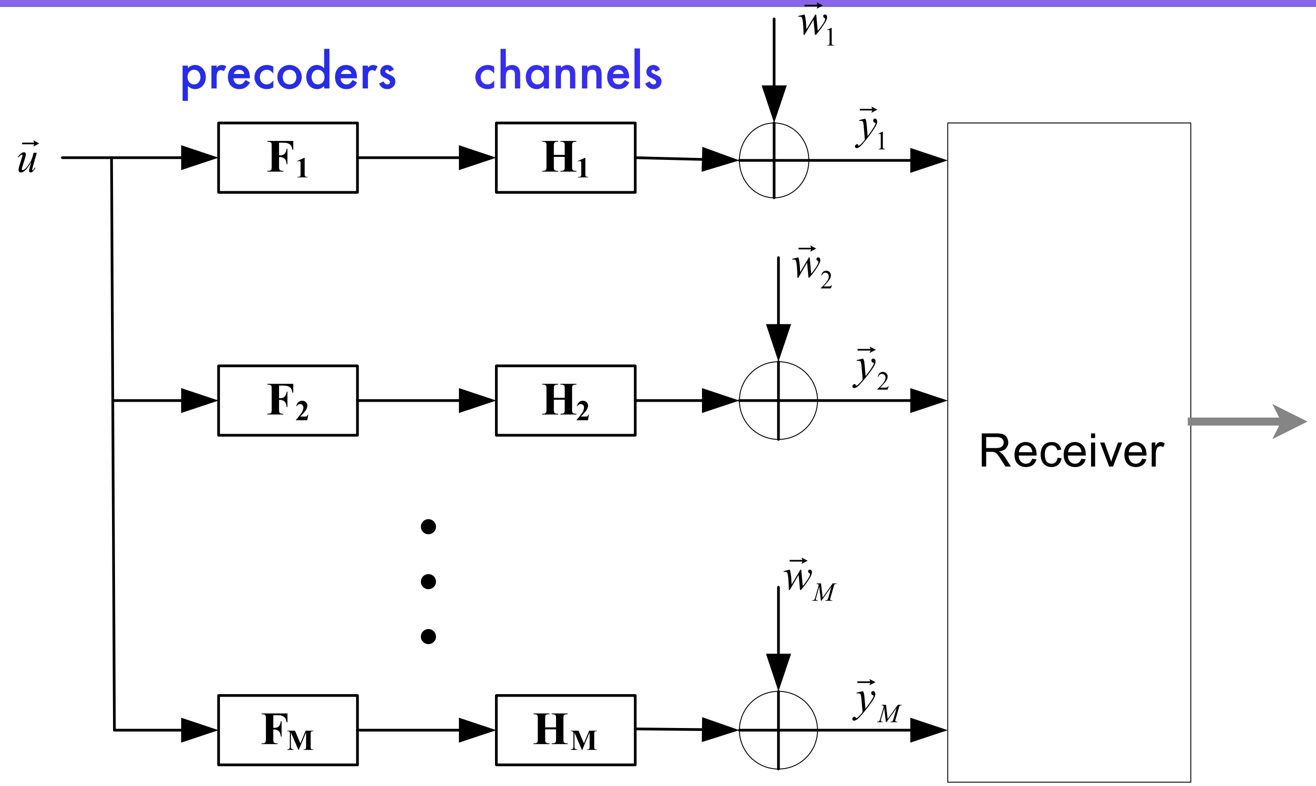
Linearly precoded H-ARQ

- Given channel knowledge, multiple retransmissions can be optimized for
  - maximum channel capacity
  - minimum MSE
- Progressive linear precoding idea [Sun-Ding-Manton]
- Consider precoded MIMO (flat fading) transmission:



$$\vec{y}_m = \mathbf{H}_m \mathbf{F}_m \vec{u} + \vec{w}_m$$

Linearly precoded H-ARQ (joint detection)



- To find successive precoders for
  - maximum channel capacity
  - minimum MSE
- Progressive linear precoding idea [Sun-Ding-Manton]

$$\vec{y} = \begin{bmatrix} \vec{y}_1 \\ \vec{y}_2 \\ \vdots \\ \vec{y}_M \end{bmatrix} = \begin{bmatrix} \mathbf{H}_1\mathbf{F}_1 \\ \mathbf{H}_2\mathbf{F}_2 \\ \vdots \\ \mathbf{H}_M\mathbf{F}_M \end{bmatrix} \vec{u} + \begin{bmatrix} \vec{w}_1 \\ \vec{w}_2 \\ \vdots \\ \vec{w}_M \end{bmatrix}$$

Optimum MMSE Precoders

$$\vec{y} = \begin{bmatrix} \vec{y}_1 \\ \vec{y}_2 \\ \vdots \\ \vec{y}_M \end{bmatrix} = \begin{bmatrix} \mathbf{H}_1\mathbf{F}_1^{opt} \\ \mathbf{H}_2\mathbf{F}_2^{opt} \\ \mathbf{H}_2\mathbf{F}_{M-1}^{opt} \\ \vdots \\ \mathbf{H}_M\mathbf{F}_M \end{bmatrix} \vec{u} + \begin{bmatrix} \vec{w}_1 \\ \vec{w}_2 \\ \vdots \\ \vec{w}_M \end{bmatrix}$$

$$\sum_{i=1}^{m-1} (\mathbf{F}_i^{\text{opt}})^H \mathbf{H}_i^H \mathbf{H}_i \mathbf{F}_i^{\text{opt}} = \mathbf{U}\mathbf{\Lambda}_{m-1}\mathbf{U}^H.$$

As a result, the MSE of the receiver output can be simplified into

$$\begin{aligned} E\{\|\hat{\vec{u}} - \vec{u}\|^2\} &= \sigma_u^2 \mathrm{Tr}\left\{(\mathbf{I}_{M_T} + \gamma\mathbf{U}\mathbf{\Lambda}_{m-1}\mathbf{U}^H + \gamma\mathbf{F}_m^H\mathbf{H}_m^H\mathbf{H}_m\mathbf{F}_m)^{-1}\right\} \\ &= \sigma_u^2 \mathrm{Tr}\left\{\mathbf{U}^H(\mathbf{I}_{M_T} + \gamma\mathbf{U}\mathbf{\Lambda}_{m-1}\mathbf{U}^H + \gamma\mathbf{F}_m^H\mathbf{H}_m^H\mathbf{H}_m\mathbf{F}_m)^{-1}\mathbf{U}\right\} \\ &= \sigma_u^2 \mathrm{Tr}\left\{(\mathbf{I}_{M_T} + \gamma\mathbf{\Lambda}_{m-1} + \gamma\mathbf{U}^H\mathbf{F}_m^H\mathbf{H}_m^H\mathbf{H}_m\mathbf{F}_m\mathbf{U})^{-1}\right\} \end{aligned}$$

Optimization Problem

$$\tilde{\mathbf{F}}_m = \mathbf{F}_m \mathbf{U},$$

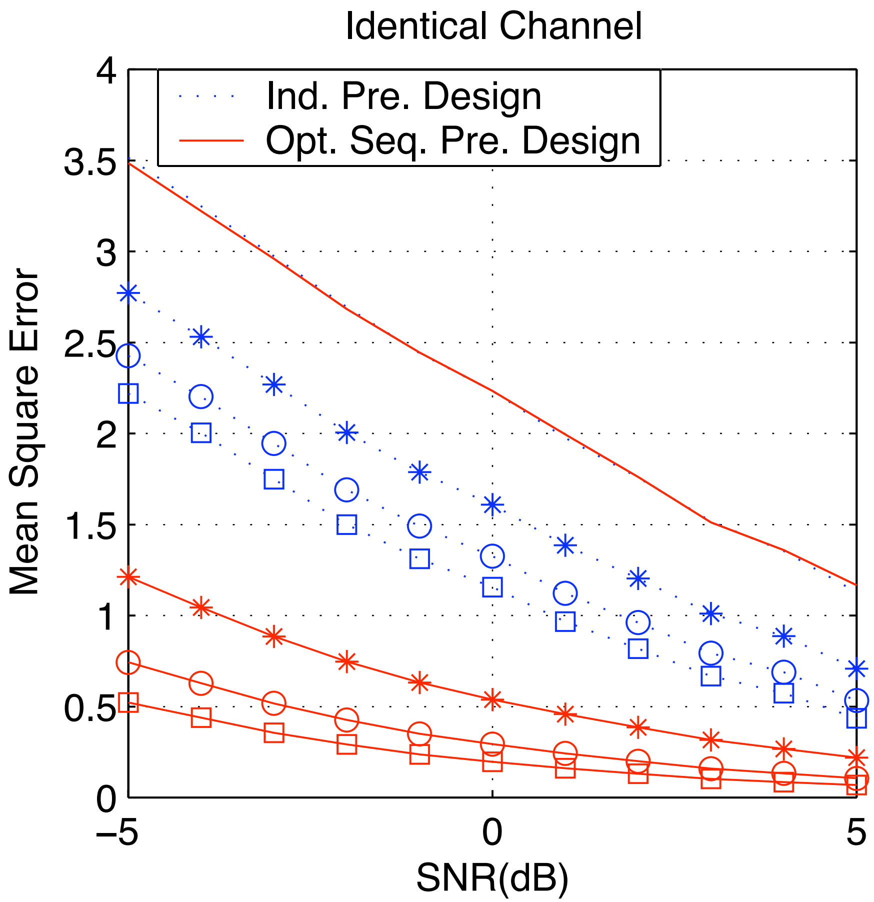
$$\arg\min_{\tilde{\mathbf{F}}_m} \ = \sigma_u^2 \mathrm{Tr}\left\{\left(\mathbf{I}_{M_T} + \gamma \mathbf{\Lambda}_{m-1} + \gamma \tilde{\mathbf{F}}_m^H \mathbf{H}_m^H \mathbf{H}_m \tilde{\mathbf{F}}_m\right)^{-1}\right\}$$

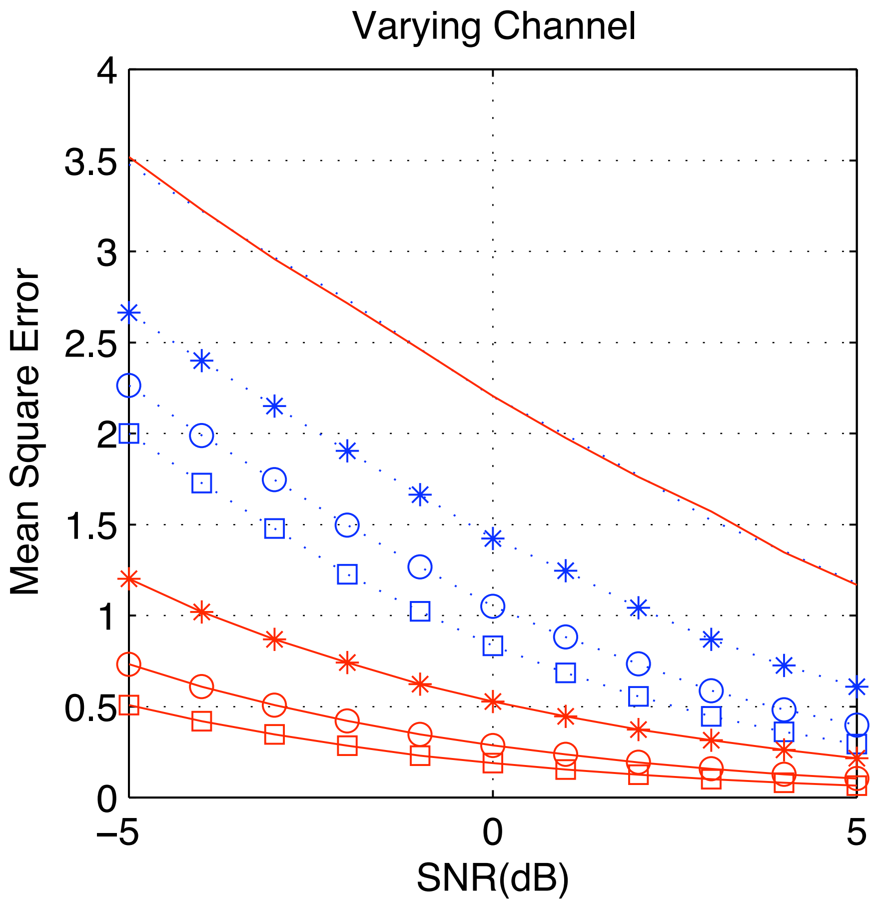
$$\text{subject to } \mathrm{Tr}\{\tilde{\mathbf{F}}_m \tilde{\mathbf{F}}_m^H\} \leq M_T.$$

- Optimal Precoder requires
  - Diagonalization
  - Reversed order pairing
  - Power constraint

Optimum MMSE MIMO-ARQ Precoding Results

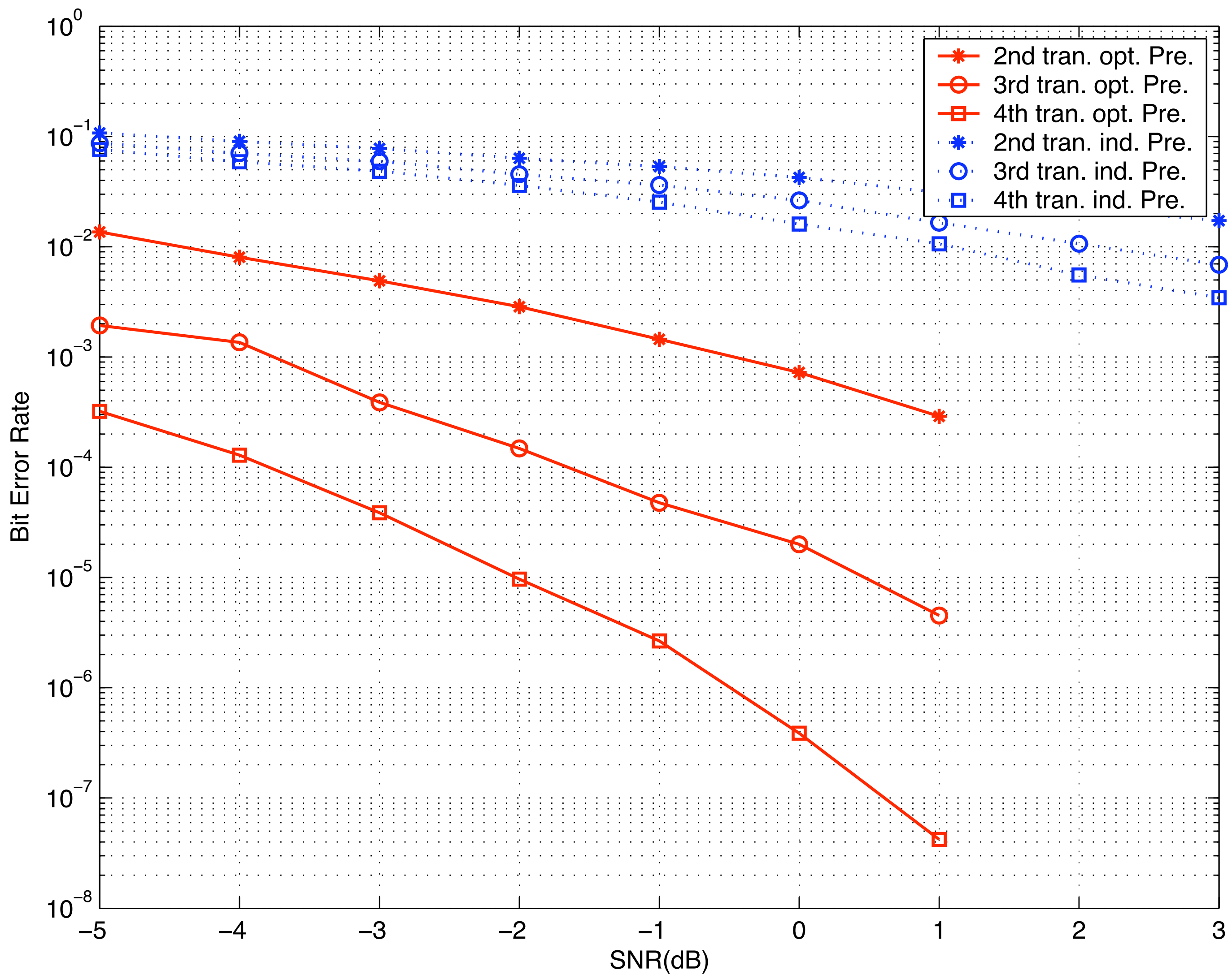
- 4x4 flat fading channels. 1000 packets/channel
- MSE results





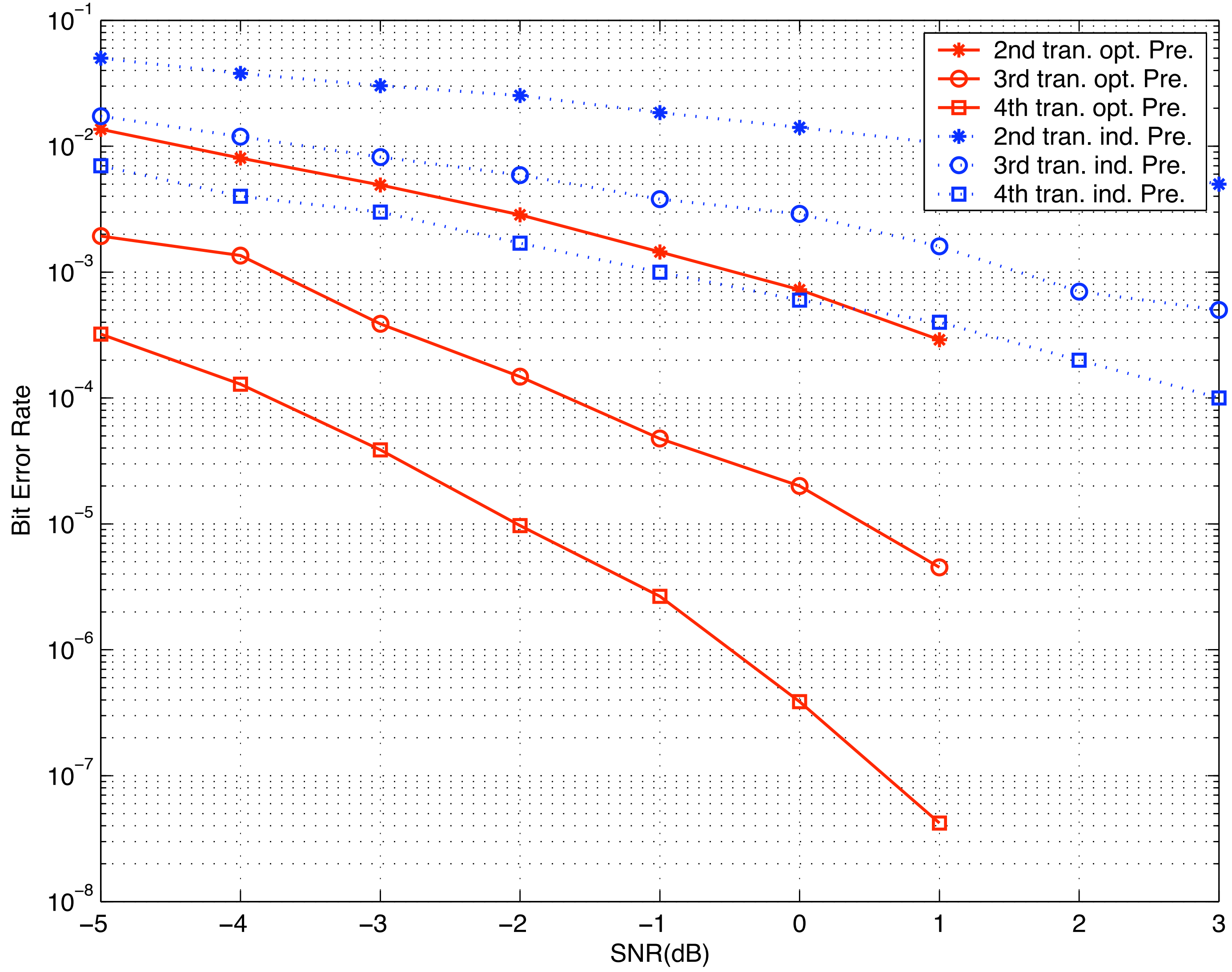
BER Results

- Comparison against simple independently optimized precoders



BER Results

- Comparison against simple independently optimized precoders +random permutation

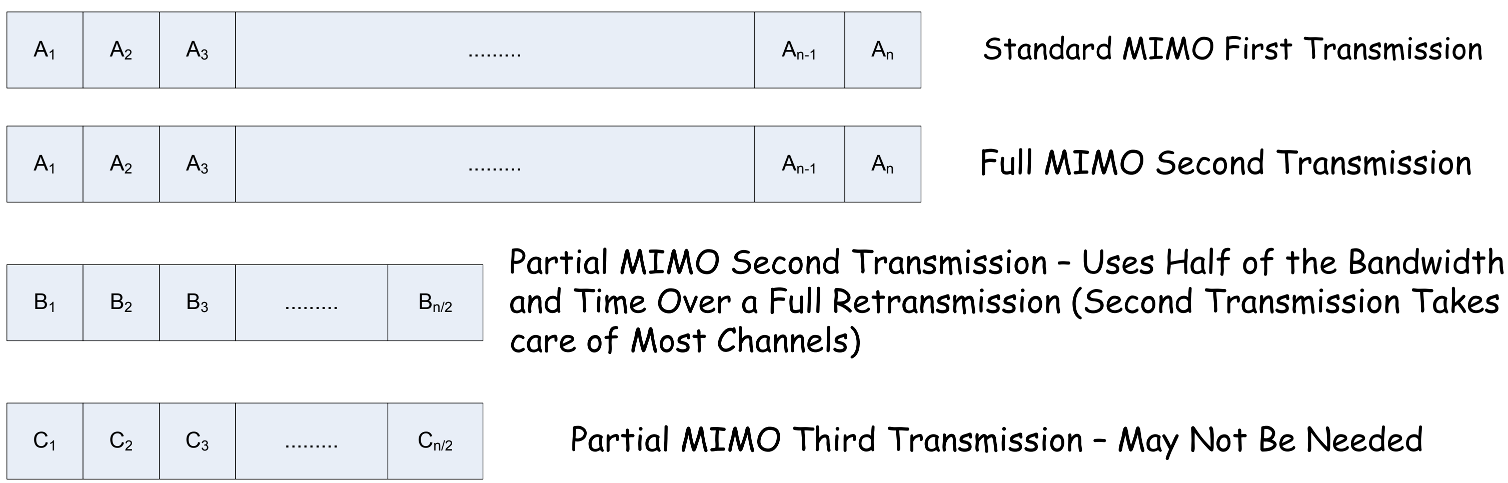


How to save bandwidth in MIMO H-ARQ?

- Hybrid ARQ can conserve bandwidth by sending partial codewords
- In MIMO Space-time coding, how to utilize H-ARQ to conserve bandwidth?
- Can space-time code be embedded or punctured like FEC?
- Partial STC retransmission during H-ARQ should be investigated.

Partial ARQ Retransmission

- Retransmission of the entire original packet multiple times is a bandwidth-wasteful strategy
- Implement partial retransmission techniques to save bandwidth and time
- With half retransmission techniques we save half of the bandwidth and time over a full retransmission



Proposed Partial ARQ Retransmission Techniques

- We will present two distinct MIMO partial ARQ (PARQ) retransmission schemes for two different channel types
  - *Fast Fading Channels*: Partial ARQ retransmission of differential orthogonal STBC - Channel knowledge not needed
  - *Slow Fading Channels*: Partial ARQ retransmission with quasi-orthogonal STBC - Channel knowledge needed
- We focus exclusively on low complexity OSTBC
- Exploiting the full diversity guaranteed by the OSTBC
- ML detector relies on inherent structure introduced by the encoder to maintain a relatively low decoding complexity.

Proposed Partial ARQ Retransmission Techniques (Continued)

- Partial ARQ retransmission of differential orthogonal STBC
  - Knowledge of the channel state information (CSI) **is not required**
  - Differential OSTBC matrices will be sent for both transmissions
  - The receiver will perform joint detection by concatenating the received matrices from both transmissions
- Partial ARQ retransmission with quasi-orthogonal STBC
  - Knowledge of the channel state information (CSI) **is required**
  - Orthogonal STBC matrices will be sent for both transmissions
  - The receiver will perform detection using the quasi-orthogonal structure of the received matrices from both transmissions

Slow Fading Channels - Problem Motivation

- Typical STBCs require perfect knowledge of the channel state information (CSI) at the receiver
- Assumption: The channel changes very slowly compared to the symbol rate, the channel matrix can be estimated fairly accurately
- Coherent detection possible at the receiver

Problem Formulation

- In the standard Alamouti scheme a codeword is assembled by taking only 2 symbols, over 2 time slots, 2 two antennas, thus x_1, x_2 form one code-block
- A standard Alamouti block is the following

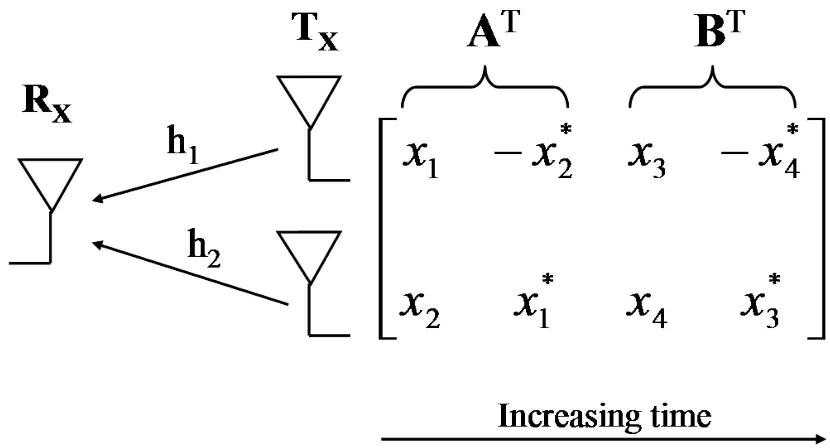
$$\mathbf{A} = \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix}$$

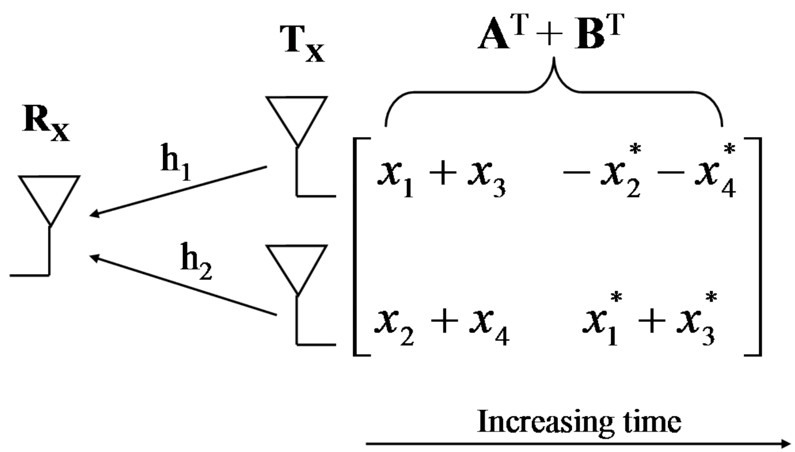
- In our formulation we need two Alamouti blocks so we form

$$\mathbf{B} = \begin{bmatrix} x_3 & x_4 \\ -x_4^* & x_3^* \end{bmatrix}$$

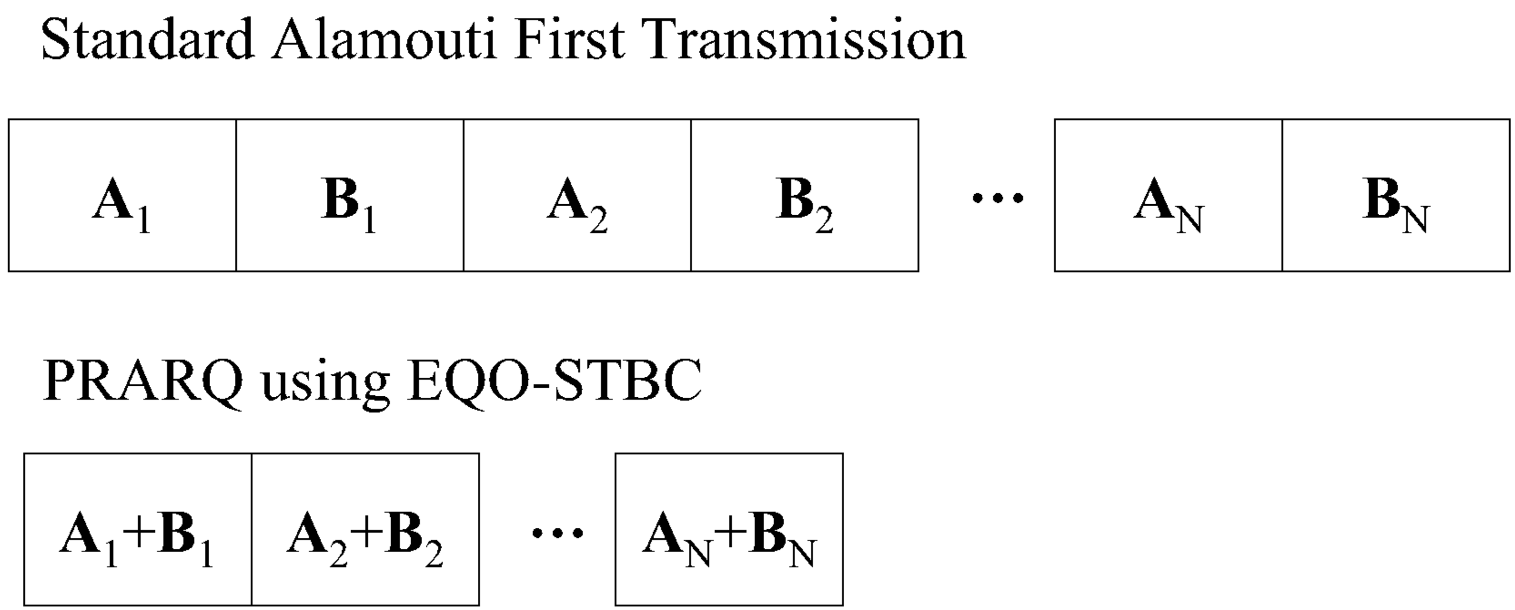
A and B are independent blocks, AND are usually retransmitted and decoded independently

PARQ Embedded QO-STBC Transmission Model





Packet Structure Model



- $\mathbf{A}$ and $\mathbf{B}$ are independent Alamouti STBC's.
- Each block represents a time width of 2 symbol baud periods.

Input Output Relationships

- The input and output equations for $\mathbf{A}$ and $\mathbf{B}$ are simply

$$\begin{bmatrix} y_{a_1} \\ y_{a_2} \end{bmatrix} = \mathbf{A} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

$$\begin{bmatrix} y_{b_1} \\ y_{b_2} \end{bmatrix} = \mathbf{B} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} n_3 \\ n_4 \end{bmatrix}$$

- Here we consider a flat fading 2×1 linear channel invariant over several space time blocks.

PARQ System Model

- For retransmission, we intentionally couple $\mathbf{A}$ and $\mathbf{B}$ and transmit them over one time block.

$$\begin{bmatrix} y_{r_1} \\ y_{r_2} \end{bmatrix} = (\mathbf{A}+\mathbf{B}) \begin{bmatrix} h_3 \\ h_4 \end{bmatrix} + \begin{bmatrix} n_5 \\ n_6 \end{bmatrix}$$

- The two transmissions are combined to form a new partial response code. This new code now depends on 4 symbols and has decoding complexity $\mathcal{O}^4$

Quasi-Orthogonal System Equation

- The code matrix formed by joining the two transmissions takes on one of the well known quasi-orthogonal forms [Jafarkhani 2000].

$$\begin{bmatrix} y_{a_1} & y_{r_1} \\ y_{a_2} & y_{r_2} \\ y_{b_1} & y_{r_1} \\ y_{b_2} & y_{r_2} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{A} \end{bmatrix} \begin{bmatrix} h_1 & h_3 \\ h_2 & h_4 \\ 0 & h_3 \\ 0 & h_4 \end{bmatrix} + \begin{bmatrix} n_1 & n_5 \\ n_2 & n_6 \\ n_3 & n_5 \\ n_4 & n_6 \end{bmatrix}$$

- Over 2 transmissions, the code represents the class of full-diversity quasi-orthogonal codes via constellation rotation
- Due to quasi-orthogonality the complexity is only $\mathcal{O}^2$.

STBC Analysis

- We can rewrite the received signal model by putting channel first, in standard form,

$$\vec{y} = \mathbf{H}\vec{x} + \vec{n} \begin{bmatrix} y_{a_1} \\ y_{a_2}^* \\ y_{b_1} \\ y_{b_2}^* \\ y_{r_1} \\ y_{r_2}^* \end{bmatrix} = \begin{bmatrix} h_1 & h_2 & 0 & 0 \\ h_2^* & -h_1^* & 0 & 0 \\ 0 & 0 & h_1 & h_2 \\ 0 & 0 & h_2^* & -h_1^* \\ h_3 & h_4 & h_3 & h_4 \\ h_4^* & -h_3^* & h_4^* & -h_3^* \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \\ n_5 \\ n_6 \end{bmatrix}$$

The channel matrix, $\mathbf{H}$, is well structured

ML Decoder

- These blocks are actually Alamouti type sub-blocks

$$\mathbf{H} = \begin{bmatrix} \mathbf{G_1} & \mathbf{0} \\ \mathbf{0} & \mathbf{G_1} \\ \mathbf{G_2} & \mathbf{G_2} \end{bmatrix}$$

- With varying assumptions and well known theory, the ML algorithm is basically an ℓ_2-norm minimization problem

$$\arg\min_{\vec{x}} \|\vec{y} - \mathbf{H}\vec{x}\|^2$$

Reduced Optimization Problem

- If we expand the ML expression we get

$$\arg\min_{\vec{x}} \left[\vec{x}^H \mathbf{H}^H \mathbf{H}\vec{x} - 2\mathbb{R}e(\vec{x}^H \mathbf{H}^H \vec{y})\right]$$

  Here we have a real linear term and a quadratic term
- For this problem to be quasi-orthogonal we need the minimization problem to take the form

$$\arg\min_{\vec{x}} f_1(x_1, x_3) + f_2(x_2, x_4).$$

- In general the 4 symbols in the quadratic term $\vec{x}^H \mathbf{H}^H \mathbf{H}\vec{x}$ cannot be decoupled. But the structure of $\mathbf{H}$ decouples symbols into 2 groups

Quasi-Orthogonality

- The quadratic matrix is

$$\mathbf{H}^H\mathbf{H} = \begin{bmatrix} \alpha+\beta & 0 & \beta & 0 \\ 0 & \alpha+\beta & 0 & \beta \\ \beta & 0 & \alpha+\beta & 0 \\ 0 & \beta & 0 & \alpha+\beta \end{bmatrix}$$

  where $\beta = |h_3|^2 + |h_4|^2$, and $\alpha = |h_1|^2 + |h_2|^2$.
- As one can see, because of the structure, we have quasi-orthogonality.
- Now that we have shown quasi-orthogonality, we extend our scheme to three total transmissions

Extension to 3 Transmissions

- The transmission scheme for the third transmission is

$$\begin{bmatrix} y_{r_1}^2 \\ y_{r_2}^2 \end{bmatrix} = (\mathbf{A} - \mathbf{B}) \begin{bmatrix} h_5 \\ h_6 \end{bmatrix} + \begin{bmatrix} n_7 \\ n_8 \end{bmatrix}.$$

$$\begin{bmatrix} y_{a_1} \\ y_{a_2}^* \\ y_{b_1} \\ y_{b_2}^* \\ y_{r_1} \\ y_{r_2}^* \\ y_{r_1}^2 \\ y_{r_2}^2 \end{bmatrix} = \begin{bmatrix} h_1 & h_2 & 0 & 0 \\ h_2^* & -h_1^* & 0 & 0 \\ 0 & 0 & h_1 & h_2 \\ 0 & 0 & h_2^* & -h_1^* \\ h_3 & h_4 & h_3 & h_4 \\ h_4^* & -h_3^* & h_4^* & -h_3^* \\ h_5 & h_6 & -h_5 & -h_6 \\ h_6^* & -h_5^* & -h_6^* & h_5^* \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \\ n_5 \\ n_6 \\ n_7 \\ n_8 \end{bmatrix}$$

Diversity Analysis

- To compute diversity we form the system model with vectorized channel,

$$\vec{y} = \mathbf{X}\vec{h} + \vec{n}$$

$$\vec{y} = \begin{bmatrix} x_1 & x_2 & 0 & 0 \\ -x_2^* & x_1^* & 0 & 0 \\ x_3 & x_4 & 0 & 0 \\ -x_4^* & x_3^* & 0 & 0 \\ 0 & 0 & x_1 + x_3 & x_2 + x_4 \\ 0 & 0 & -x_2^* - x_4^* & x_1^* + x_3^* \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \end{bmatrix} + \vec{n}$$

- The error codeword matrix is $\mathbf{X} - \hat{\mathbf{X}}$, and consequently the squared error matrix is

$$\mathbf{\Gamma}(\mathbf{X}, \hat{\mathbf{X}}) = (\mathbf{X} - \hat{\mathbf{X}})^H (\mathbf{X} - \hat{\mathbf{X}})$$

Code Analysis

- The performance of a space-time code is determined by manipulating $\mathbf{\Gamma}(\mathbf{X}, \hat{\mathbf{X}})$ [Tarokh et al, 98].
- Denote the set of eigenvalues of $\mathbf{\Gamma}(\mathbf{X}, \hat{\mathbf{X}})$ as $\{\lambda_i^r\}_{i=1}$.
- The three most well-known ST code performance criterion are to maximize the following functions
- Minimum rank r of the matrix $\mathbf{\Gamma}(\mathbf{X}, \hat{\mathbf{X}})$ over all pairs of distinct codewords
- Minimum determinant, $\prod_{i=1}^{r} \lambda_i$, over all pairs of distinct codewords
- Minimum trace, $\sum_{i=1}^{r} \lambda_i$, over all possible distinct codewords
- The first criterion is by far the most important and is referred to as the code diversity

Constellation Rotation

- Original quasi-orthogonal codes do not have full diversity unless constellation rotation are used [Sharma and Papadias 03].
- Without rotation, the original source vector is,

$$\vec{x} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}^T$$

but when using rotation the source vector is

$$\vec{\overline{x}} = \begin{bmatrix} x_1 & x_2 & \overline{x}_3 & \overline{x}_4 \end{bmatrix}^T.$$

- In this case, $x_1, x_2 \in \mathcal{A}$, while $\overline{x}_1, \overline{x}_2 \in \overline{\mathcal{A}}$. Because of rotation, $\overline{\mathcal{A}} = e^{j\theta}(\mathcal{A})$
The optimal rotation angle for QPSK was found to be $.53$ radians.
Our new receiver is $\arg\min_{\vec{\overline{x}}} f_1(x_1, \overline{x}_3) + f_2(x_2, \overline{x}_4)$.

Diversity with Constellation Rotation

- With constellation rotation, our new code matrix has the following form

$$\vec{y} = \begin{bmatrix} x_1 & x_2 & 0 & 0 \\ -x_2^* & x_1^* & 0 & 0 \\ \overline{x}_3 & \overline{x}_4 & 0 & 0 \\ -\overline{x}_4^* & \overline{x}_3^* & 0 & 0 \\ 0 & 0 & x_1 + \overline{x}_3 & x_2 + \overline{x}_4 \\ 0 & 0 & -x_2^* - \overline{x}_4^* & x_1^* + \overline{x}_3^* \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \end{bmatrix} + \vec{n}$$

This code matrix, $\mathbf{X}$, has diversity order 4, and is thus full diversity.

Summary of Code Analysis

	diversity	min dtrmnt	min trace
Alamouti Tx2	4	1	4
PRARQ Tx2	4	0.28	4
QO-STBC Tx1	4	.975	4
Alamouti Tx3	6	1	6
PRARQ Tx3	6	1	6

Table: PRARQ EQO-STBC design results for QPSK symbols with constellation rotation.

According to the table, our PARQ Tx2 code does reasonably well overall, and PARQ Tx3 has the same code metrics as the orthogonal scheme.

Channel Dependence on Performance

- Now that the average code performance has been analyzed formally, it is important to look at specific channel dependence on performance.
- Taking the SVD of $\mathbf{H}$ we get

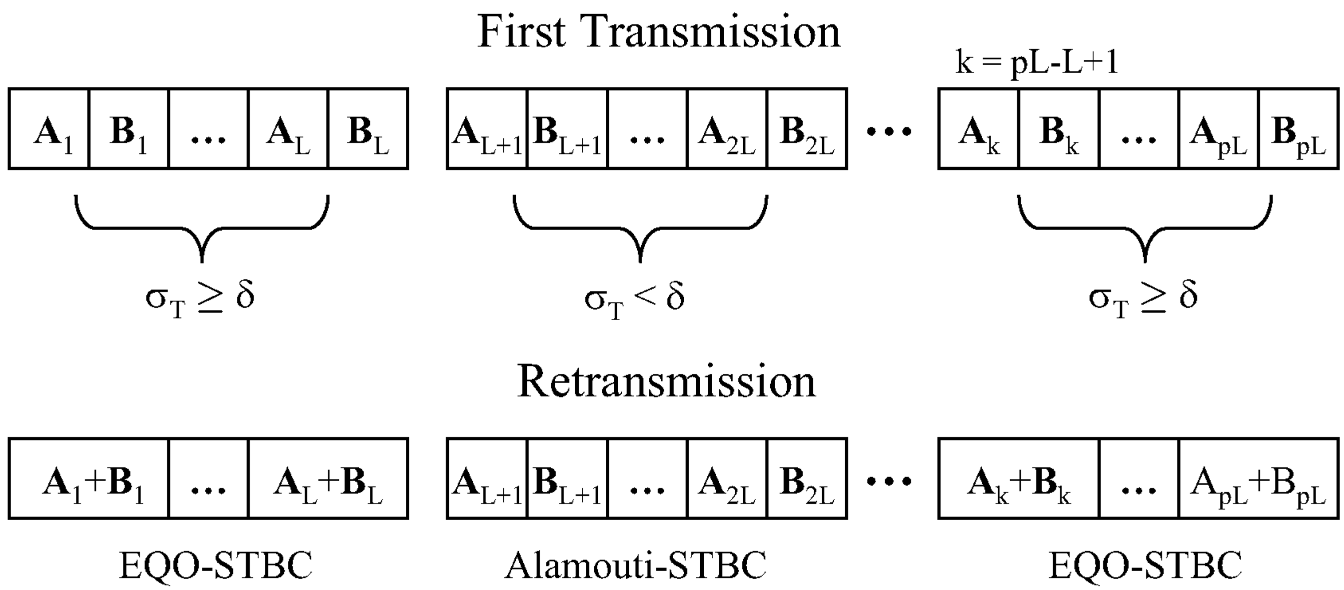
$$diag(\mathbf{\Sigma}_H) = \begin{bmatrix} \sqrt{\alpha+\beta} \\ \sqrt{\alpha+\beta} \\ \sqrt{\alpha} \\ \sqrt{\alpha} \end{bmatrix} = \begin{bmatrix} \sqrt{|h_1|^2+|h_2|^2+|h_3|^2+|h_4|^2} \\ \sqrt{|h_1|^2+|h_2|^2+|h_3|^2+|h_4|^2} \\ \sqrt{|h_1|^2+|h_2|^2} \\ \sqrt{|h_1|^2+|h_2|^2} \end{bmatrix}$$

- At high SNR, the effective SNR is proportional to $\frac{\sigma_{min}^2}{\sigma_n^2}$.
- Because the retransmission, $\mathbf{A}+\mathbf{B}$, cannot be decoupled without reliable information from the 1^{st} transmission, we expect $\sigma_{min}^2 = |h_1|^2 + |h_2|^2$.
- We denote $\sigma_T = \frac{\sigma_{min}^2}{\sigma_n^2}$.

Refined ARQ Feedback Request

- Because $\sigma_{min}^2 = |h_1|^2 + |h_2|^2$ instead of $|h_1|^2 + |h_2|^2 + |h_3|^2 + |h_4|^2$, we propose a simple, modified feedback repeat request.
- Bad performance is closely related σ_T, and this quantity can be easily estimated by the receiver.
- based on experimental tradeoffs, the receiver can either request a full or half retransmission depending on the value of σ_T.
- For a particular sub-packet, requesting a full or half retransmission only requires one bit of information.

Modified Repeat Request Diagram



- The packet is first divided into sub-packets where the channel is constant.
- The receiver evaluates each sub-packet, and either requests a full or half retransmission based on the value of σ_T.
- δ, is a chosen threshold that facilitates the trade-off between performance and bandwidth efficiency.

PARQ Simulation Results without Constellation Rotation

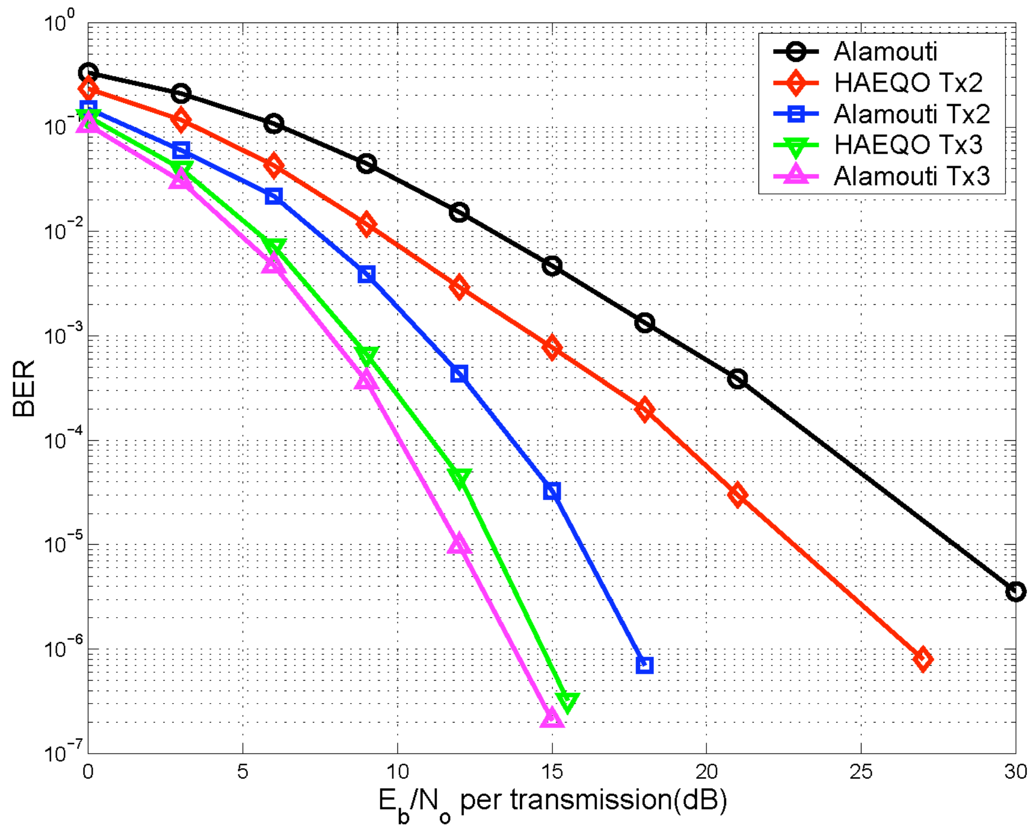


Figure: Hybrid ARQ STBC performances under Rayleigh fading for 1, 2, and 3 total transmissions without constellation rotation.

PARQ Performance Results with Constellation Rotation

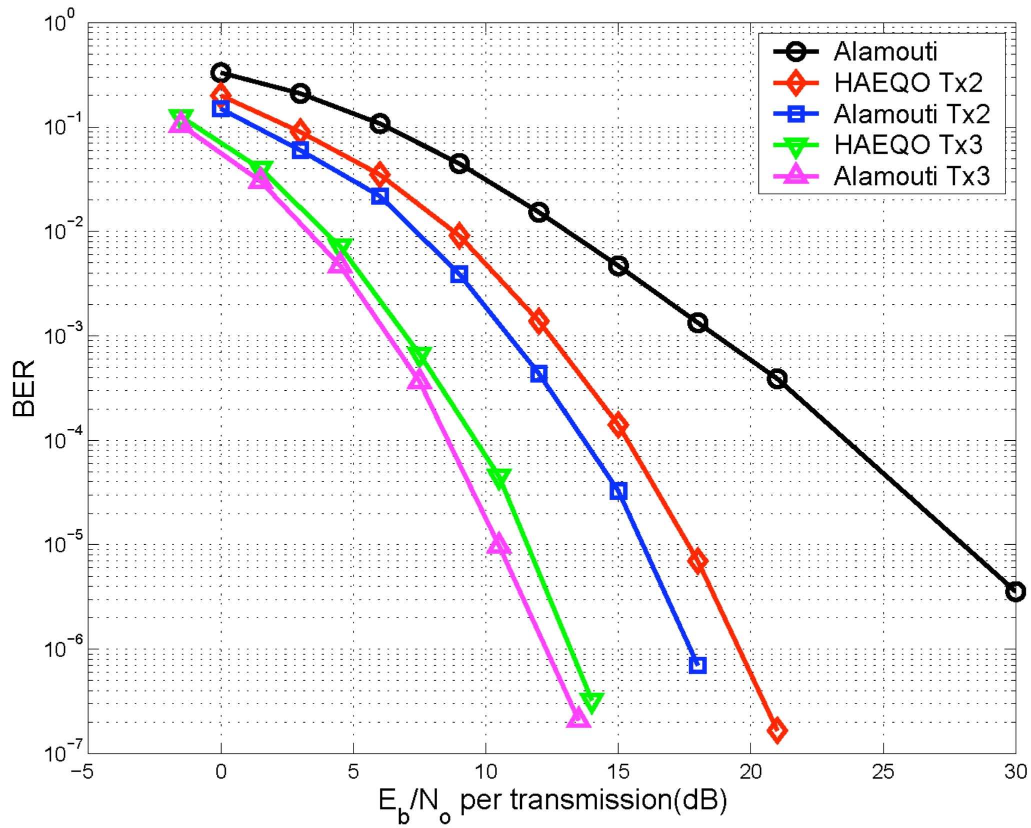


Figure: Hybrid ARQ STBC performances under Rayleigh fading for 1, 2, and 3 total transmissions using constellation rotation

Summary of Simulation Results

- The previous plot did not utilize the modified repeat request
- Although 50 percent of the bandwidth was saved, the performance penalty was 2 and 1 dB for one and two retransmissions respectively.
- The next slide shows the performance penalty when the bandwidth saving is 40 percent

Modified Repeat Request Simulation

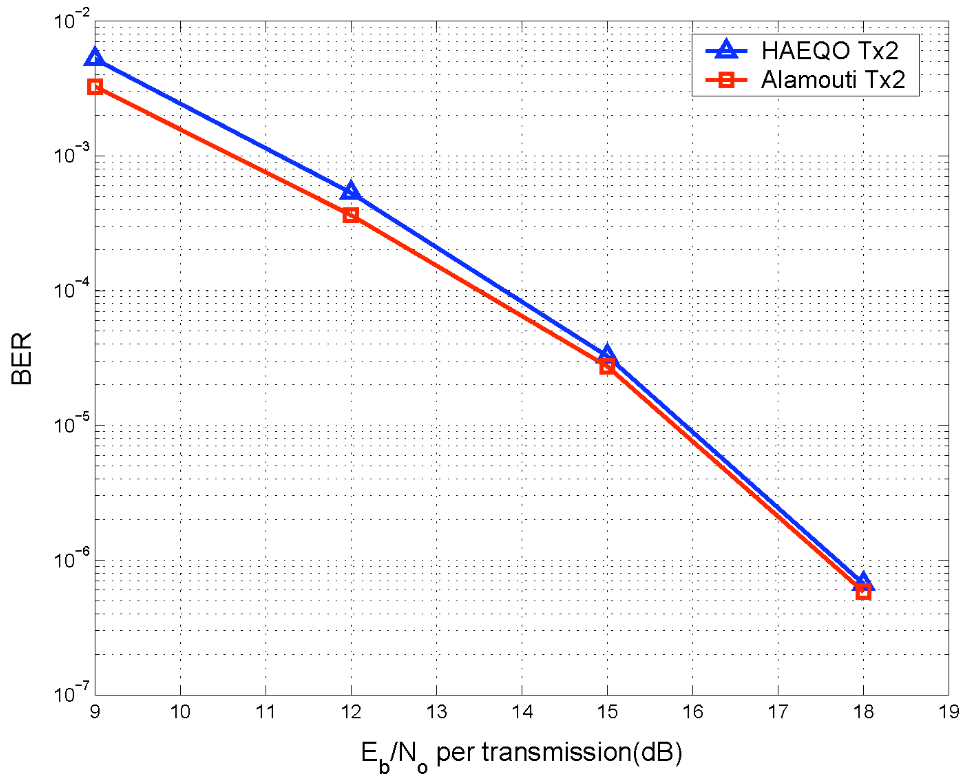


Figure: Comparisons of our scheme (w/ rotation) with standard Alamouti transmission with a bandwidth savings of 40 percent

Summary

- Integration of H-ARQ and MIMO brings great performance gain.
- Simply mapping diversity enhances joint detection.
- Progressive H-ARQ precoding optimizes capacity.
- Partial response H-ARQ STC retransmission saves bandwidths based on detection reliability
- Partial H-ARQ retransmission controllable via soft decoder LLR.
- Exciting possibilities involving various means of MIMO-ARQ integration and detection.