Homework 2

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CES 500
3.3.05

1.4 Want $W_q = 75\ s$ **Find:** # queue slots needed (size of waiting room)

$$\lambda = 3\ \frac{\text{call}}{\text{min}} \cdot \frac{1\ \text{min}}{60\ s} = \frac{1}{20}\ s^{-1}$$

$$L_q = \lambda W_q = \frac{1}{20} \cdot 75 = \frac{15}{4}$$

Double this so that overflow can be accomodated

$\Rightarrow$ $\boxed{8 \text{ persons}}$ allowed in waiting room

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1.6 Next few pages show the raw data, processed into the quantities used here.

$$L = \frac{1(67) + 2(49) + 3(6) + 0(16)}{137} = \frac{173}{137} \approx \boxed{1.3}$$

(a) $$W_q = \frac{0+2+0+3+8+11+6+9+6+7+6(0)+3+4+2+2}{20} = \boxed{3.15}$$

$$W = \frac{3+9+9+12+18+15+14+14+11+10+6+3+5+4+9+9+11+10+10+5}{20}$$

$$W = \frac{187}{20} = \boxed{9.35}$$

(b) $$W_{waiters} = \frac{9+12+18+15+14+14+11+10+11+10+10+5}{12} = \boxed{11.6}$$

$$L_q = \frac{1(49)+2(6)}{137} = \boxed{0.45}$$

$$\text{Idle fraction} = \frac{14}{137} = \boxed{0.10}$$

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wait	cust	interarrival	service
0	1	1	3
2	2	9	7
0	3	6	9
3	4	4	9
8	5	7	10
11	6	9	4
6	7	5	8
9	8	8	5
6	9	4	5
7	10	10	3
0	11	6	6
0	12	12	3
0	13	6	5
0	14	8	4
0	15	9	9
0	16	5	9
3	17	7	8
4	18	8	6
2	19	8	8
2	20	7	3

t	in service	join	# system	# system
0	0		0	0
1	1	1	1	1
2	1	2	2	2
3	1			2
4	2		1	1
5	2			1
6	2			1
7	2			1
8	2			1
9	2			1
10	2			1
11	3	3		1
12	3			1
13	3			1
14	3			1
15	3			1
16	3			1
17	3	4	2	2
18	3			2
19	3			2
20	4		1	1
21	4	5	2	2
22	4			2
23	4			2
24	4			2
25	4			2
26	4			2
27	4			2
28	4	6	3	3
29	5		2	2
30	5			2
31	5			2
32	5			2
33	5			2
34	5			2
35	5			2
36	5			2
37	5	7	3	3
38	5			3
39	6		2	2
40	6			2
41	6			2
42	6	8	3	3
43	7		2	2
44	7			2
45	7			2
46	7			2
47	7			2
48	7			2
49	7			2
50	7	9	3	3

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51	8		2	2
52	8			2
53	8			2
54	8	10	3	3
55	8			3
56	9		2	2
57	9			2
58	9			2
59	9			2
60	9			2
61	10		1	1
62	10			1
63	10			1
64	11	11		1
65	11			1
66	11			1
67	11			1
68	11			1
69	11			1
70	12	12		1
71	12			1
72	12			1
73			0	0
74				0
75				0
76				0
77				0
78				0
79				0
80				0
81				0
82	13	13	1	1
83	13			1
84	13			1
85	13			1
86	13			1
87			0	0
88	14	14	1	1
89	14			1
90	14			1
91	14			1
92			0	0
93				0
94				0
95				0
96	15	15	1	1
97	15			1
98	15			1
99	15			1
100	15			1
101	15			1
102	15			1

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103	15			1
104	15			1
105	16	16		1
106	16			1
107	16			1
108	16			1
109	16			1
110	16	17	2	2
111	16			2
112	16			2
113	17		1	1
114	17			1
115	17			1
116	17			1
117	17	18	2	2
118	17			2
119	17			2
120	17			2
121	18		1	1
122	18			1
123	18			1
124	18			1
125	18	19	2	2
126	18			2
127	19		1	1
128	19			1
129	19			1
130	19			1
131	19			1
132	19			1
133	19	20	2	2
134	19			2
135	20		1	1
136	20			1
137	20			1
138			0	0
139				
140				
141				
142				
143				
144				
145				
146				
147				
148				

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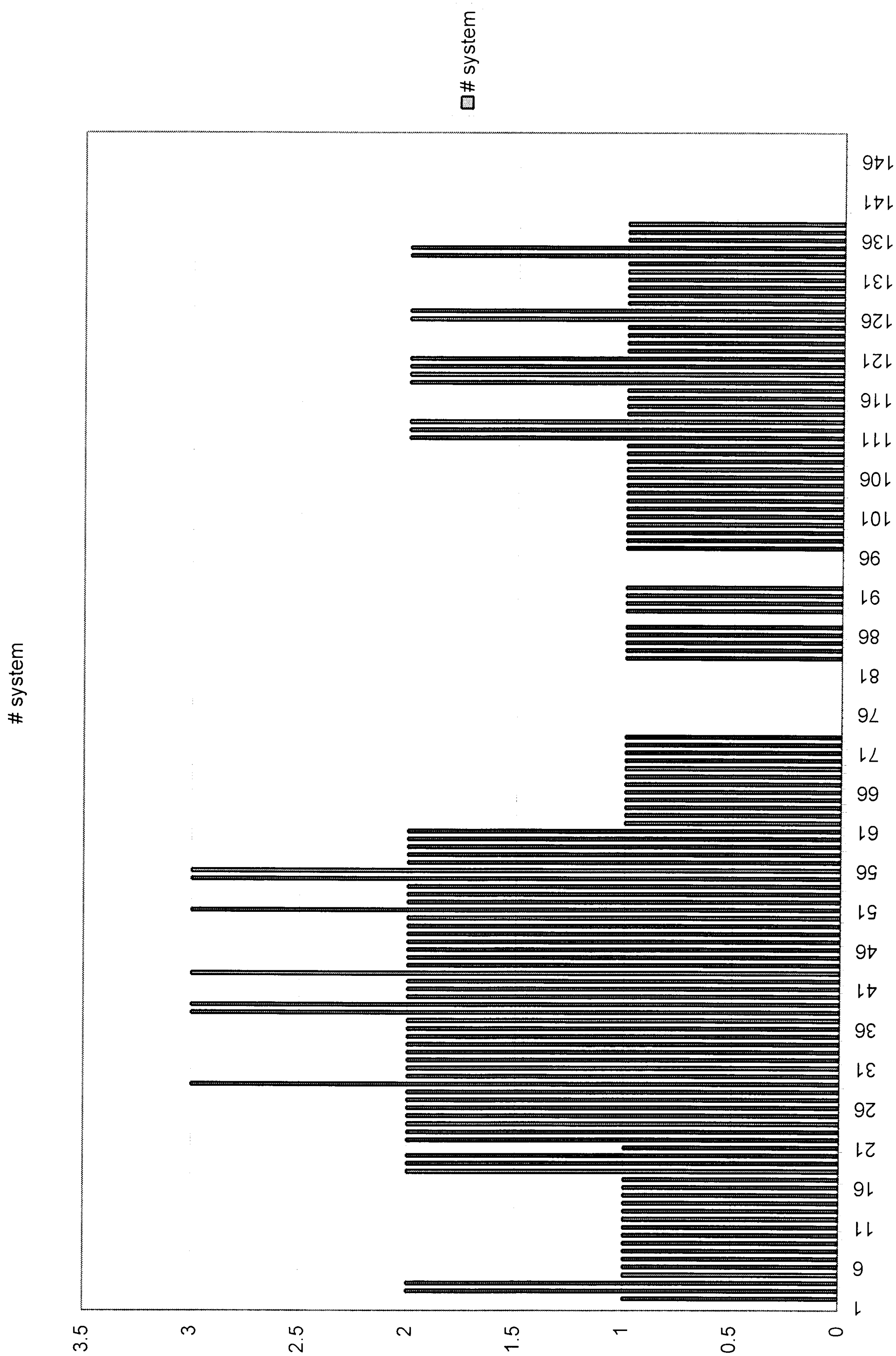

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3/2/2005

ID	arrive	start	complete	service	1/μ
1	5	5	7	2	4.6
2	10	10	17	7	
3	15	17	23	6	
4	20	23	29	6	
5	25	29	35	6	
6	30	35	38	3	
7	35	38	39	1	
8	40	40	44	4	
9	45	45	46	1	
10	50	50	60	10	

t	served	queue
5	1	
7		
10	2	
15	2	3
17	3	
20	3	4
23	4	
25	4	5
29	5	
30	5	6
35	6	7
38	7	
39		
40	8	
44		
45	9	
46		
50	10	
60		

L
1.1
(2+5+2*2+3+2*3+2+2*4+1+2*5+2*3+1+4+1+10)/60

%idle
23
100*(5+3+1+1+4)/60

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Attempt at #1.7c using package

M/M/1: POISSON ARRIVALS TO A SINGLE EXPONENTIAL SERVER

Input Parameters:

Arrival rate (λ)	0.2
Mean service time ($1/\mu$)	4.6

Plot Parameters:

Maximum size for probability chart	20
Total time horizon for probability plotting	60.

correct inputs I think

Results:

Mean interarrival time ($1/\lambda$)	5
Service rate (μ)	0.217391304
Server utilization (ρ)	92.00%
Mean number of customers in the system (L)	11.5
Mean number of customers in the queue (Lq)	10.58
Expected non-empty queue size (Lq')	12.5
Mean waiting time (W)	57.5
Mean waiting time in the queue (Wq)	52.9
Mean length of busy period (B)	57.5

wrong output

Is this because we don't have Poisson arrival ~~time~~ process? but rather a uniform arrival time distribution?

Also we don't know that the server is "exponential"?

HW 2 Worksheet

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1. (a) $P(X < Y) = \boxed{\frac{1}{2}}$ because memoryless

(b) $P(Z < X) = P(Z < X \wedge X < Y) + P(Z < X \wedge Y < X)$

$= P(Z < X \mid X < Y)P(X < Y) + P(Z < X \mid Y < X)\, P(Y < X)$

$= 0\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \boxed{\frac{1}{4}}$

(c) For X, (system wait) = (service time) because no queue

cdf: $f(t) = P(T_w \le t) = P(T_s \le t) = \begin{cases} 1 - e^{-\mu t}, & t \ge 0 \\ 0 & \text{o.w.} \end{cases}$

Since $\mu = \frac{1}{15}$, $\boxed{f(t) = \begin{cases} 1 - e^{-1/15 t}, & t \ge 0 \\ 0 & \text{o.w.} \end{cases}}$

$W = E(T_w) = E(T_s) = \frac{1}{\mu} = \boxed{15 \text{ min.}}$

(d) Same as (c)

(e) have to do this later

$E(T_{w_Z}) = \underbrace{E(T_{w_{XY}})}_{\text{wait for X, Y}} + \underbrace{E(T_{s_Z})}_{\text{serve Z}}$?