David Bozarth
CES 400
9.20.05

HW 3

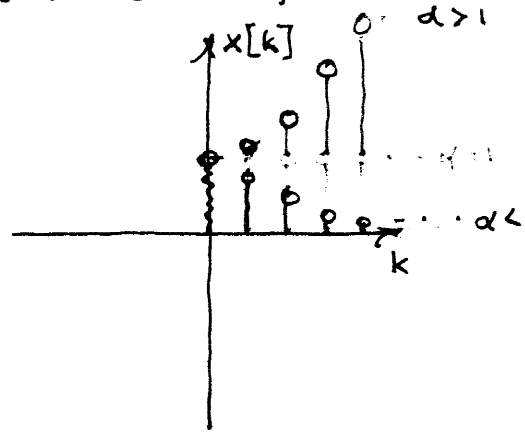
2.21 a

$x[n] = \alpha^n u[n]$

$h[n] = \beta^n u[n]$

Find $y[n] = x[n] * h[n]$

First, consider $x[k]$ and $h[n-k]$ as function of k:

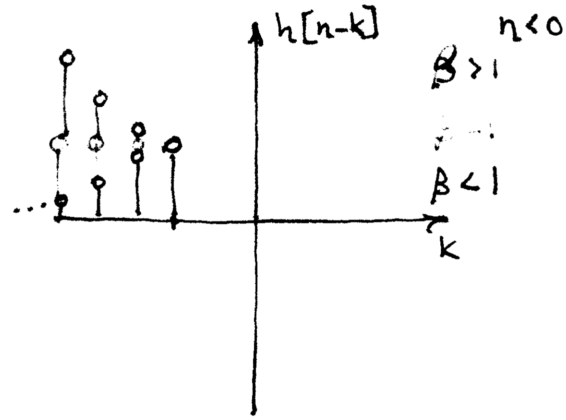


$k < 0 \Rightarrow x[k] = 0$; $k \geq 0 \Rightarrow x[k] = \alpha^k$

$n < 0 \Rightarrow x[k]\, h[n-k] = 0$

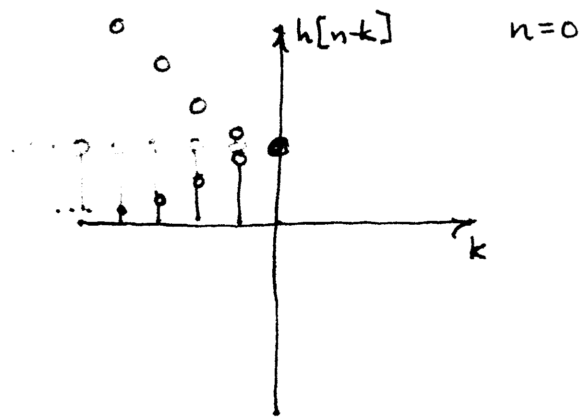
$n \geq 0 \Rightarrow (0 \leq k \leq n \Rightarrow x[k]\, h[n-k]$ nonzero$)$

$$\Rightarrow y[n] = \sum_{k=0}^{n} \alpha^k h[n-k] = \sum_{k=0}^{n} \alpha^k \beta^{n-k}$$

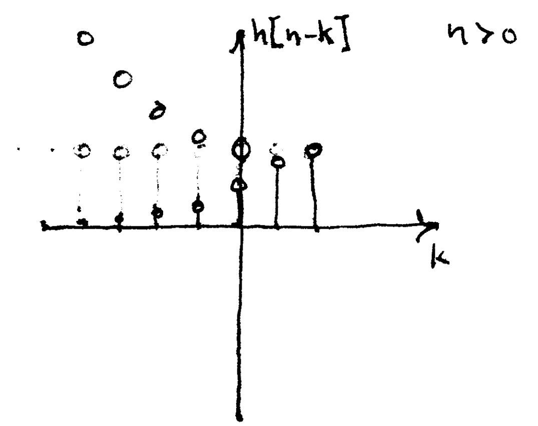


$$y[n] = \sum_{k=0}^{n} \frac{\alpha^k}{\beta^k} \cdot \beta^n = \beta^n \left(\frac{1 - \left(\frac{\alpha}{\beta}\right)^{n+1}}{1 - \frac{\alpha}{\beta}} \right) ; \; n \geq 0$$

$$y[n] = \begin{cases} \dfrac{\beta^{n+1} - \alpha^{n+1}}{\beta - \alpha} & ; \; n \geq 0 \\ 0 & ; \; n < 0 \end{cases}$$



$$y[n] = \boxed{\frac{\beta^{n+1} - \alpha^{n+1}}{\beta - \alpha} u[n]}$$

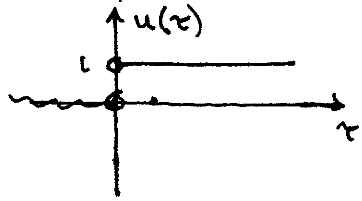


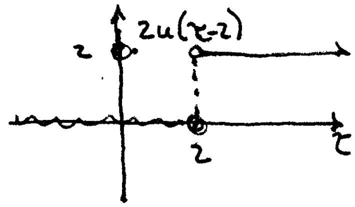
D. Bozarth

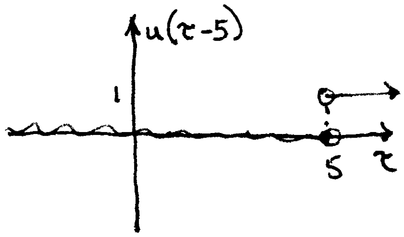
2.22 (b) $x(t) = u(t) - 2u(t-2) + u(t-5)$

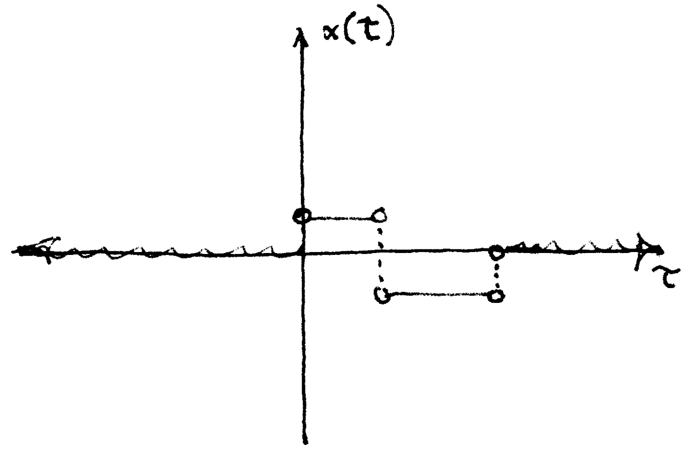
$$h(t) = e^{2t}\,u(1-t)$$

Graph $x(\tau)$:

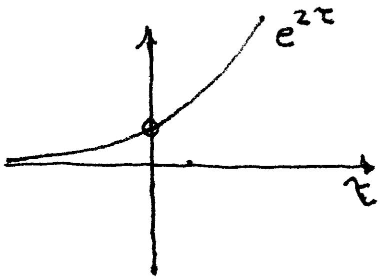


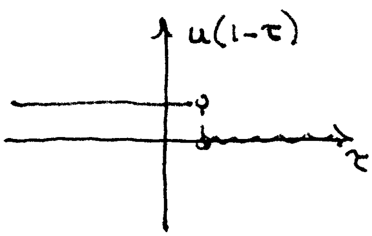


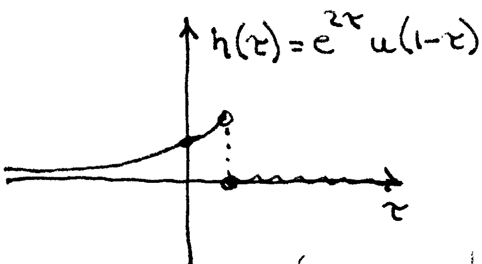




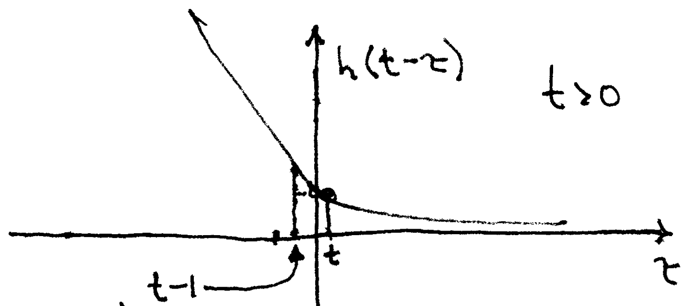
Graph $h(t-\tau)$:

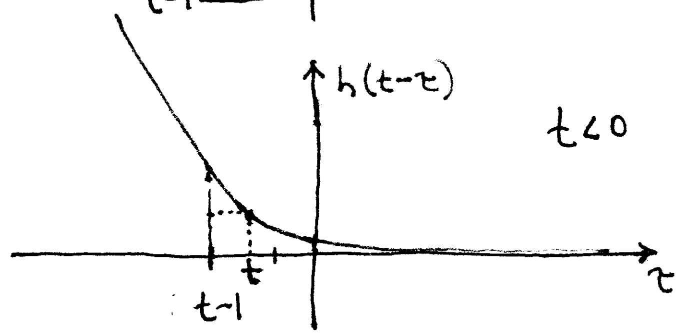






See next page...





When $t-1<0$:

Since $h(t-\tau) = e^{-2\tau}\cdot u(t-1)$

$$y(t) = \int_0^2 1\cdot h(t-\tau)\,d\tau - \int_2^5 1\cdot h(t-\tau)\,d\tau$$

$$\boxed{y(t) = \int_0^2 e^{2(t-\tau)}\,d\tau - \int_2^5 e^{2(t-\tau)}\,d\tau \;;\; t<1}$$

cont'd 2 pages later

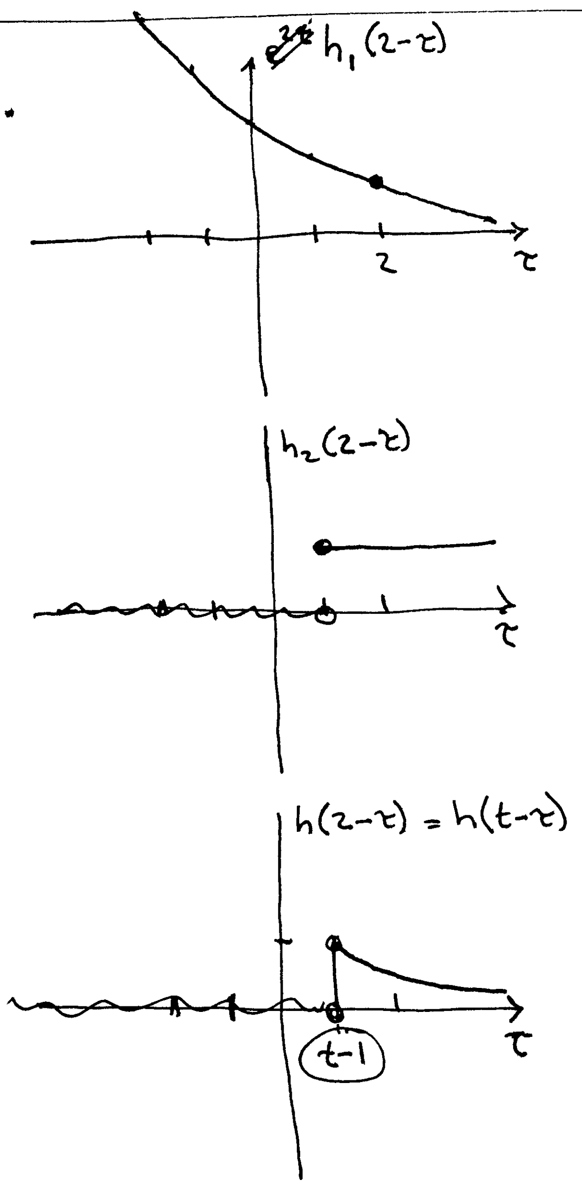
$h(\tau) = e^{2\tau} \cdot u(1-\tau)$ … $\boxed{\text{what is } h(t-\tau)\text{ ?}}$

D. Bozarth

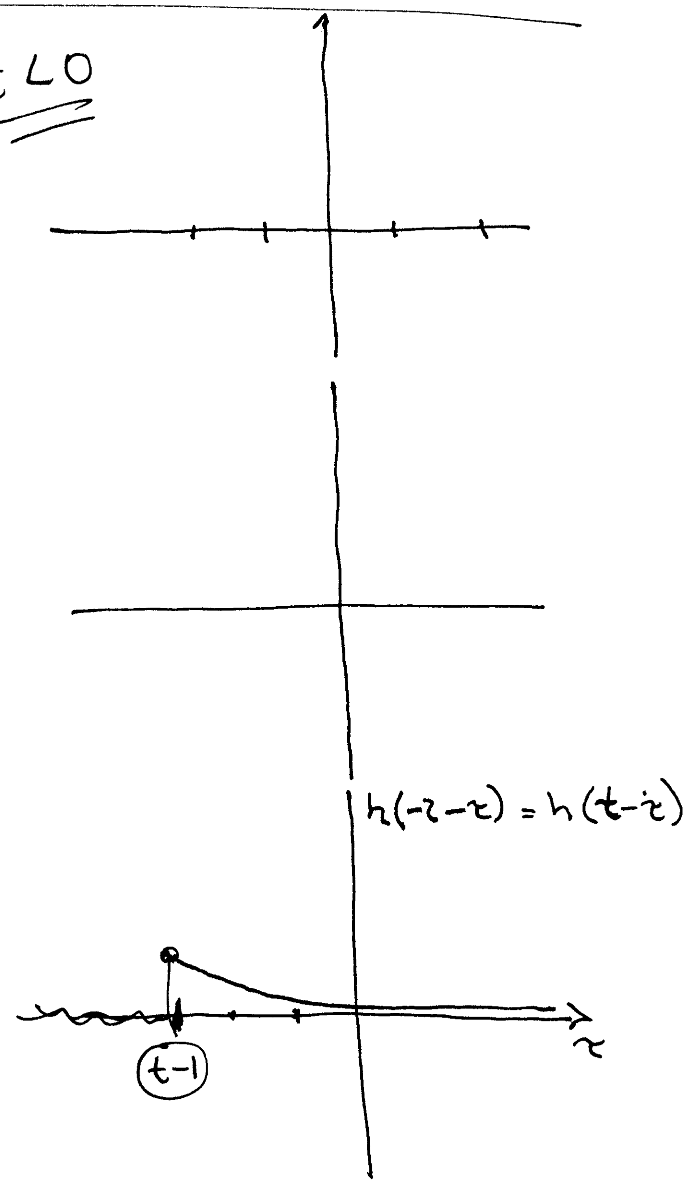
$h_1(\tau) = e^{2\tau}$ $\qquad h_2(\tau) = u(1-\tau)$ $\qquad \boxed{\text{Using } t=2 \text{ as example}}$

τ_0	$h_1(\tau_0)$	$h_2(\tau_0)$	τ when $\tau_0 = 2-\tau$	$h_1(2-\tau)$	$h_2(2-\tau)$
-2	e^{-4}	$u(3)=1$	4	e^{8}	$u(1-4)=0$
-1	e^{-2}	$u(2)=1$	3	e^{6}	$u(1-3)=0$
0	1	$u(1)=1$	2	e^{4}	$u(1-2)=0$
1	e^{2}	$u(0)=1$	1	e^{2}	$u(0)=1$
2	e^{4}	$u(-1)=0$	0	1	$u(1)=1$

$t > 0$

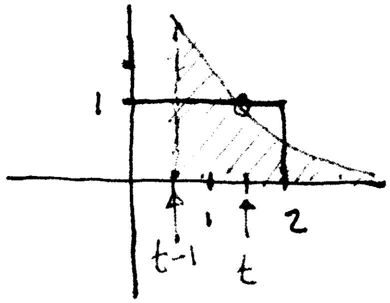


$t < 0$



D. Buzarth

when $0 < t-1 \leq 2$:



$$\boxed{y(t) = \int_{t-1}^{2} e^{2(t-\tau)}\, d\tau - \int_{2}^{5} e^{2(t-\tau)}\, d\tau \; ; \quad 1 < t \leq 3}$$

when $2 < t-1 \leq 5$:

$$\boxed{y(t) = -\int_{t-1}^{5} e^{2(t-\tau)}\, d\tau \; ; \quad 3 < t \leq 6}$$

When $5 < t-1$:

$$\boxed{y(t) = 0 \; ; \quad 6 < t}$$

I started evaluating the first integral and got as far as

$$y(t) = -\frac{1}{2} e^{2t}\left(e^{-4} - e^{-2(t-1)} - 5\left(e^{-10} - e^{-4}\right)\right)$$

no time left