

2.49

Show: If LTI with $h[n]$ is stable, then

$$\sum_{k=-\infty}^{\infty} |h[k]| < \infty.$$

Proof by Contrapositive

Suppose there is an LTI with $h[n]$ such that

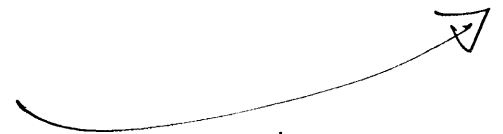
$$\sum_{k=-\infty}^{\infty} |h[k]| = \infty.$$

Consider the input $x[n] = \begin{cases} 1 & \text{if } h[-n] > 0 \\ 0 & \text{if } h[-n] = 0 \\ -1 & \text{if } h[-n] < 0 \end{cases}$

Note $|x[n]| \leq 1$ for all n .

$$\text{Then } y[n] = \sum_{k=-\infty}^{\infty} h[k] x[n-k]$$

$$y[0] = \sum_{k=-\infty}^{\infty} h[k] x[-k]$$

next
page 

2.49 (b) cont'd ~~the~~.

$$y[0] = \sum_{k=-\infty}^{\infty} h[k] x[-k]$$

$$\text{if } \begin{cases} h[k] < 0 \Rightarrow x[-k] = -1 \Rightarrow h[k] x[-k] = -h[k] > 0 \\ h[k] = 0 \Rightarrow x[-k] = 0 \Rightarrow h[k] x[-k] = 0 = h[k] \\ h[k] > 0 \Rightarrow x[-k] = 1 \Rightarrow h[k] x[-k] = h[k] > 0 \end{cases}$$

~~Thus if any $h[k] < 0$, then~~
 ~~$|y[0]| < \sum_{k=-\infty}^{\infty} |h[k] x[-k]|$~~

Thus $h[k] x[-k] \geq 0$, for all k .

$$\text{Then } |y[0]| = \left| \sum_{k=-\infty}^{\infty} h[k] x[-k] \right| = \sum_{k=-\infty}^{\infty} |h[k] x[-k]|$$

$$|y[0]| = \sum_{k=-\infty}^{\infty} |h[k]| |x[-k]| = \sum_{k=-\infty}^{\infty} |h[k]|$$

$\therefore |y[0]| = \infty$ unstable because bounded input gives unbounded output.

#