D. Bozarth

2.49 Show: If LTI with $h[n]$ is stable, then

$$\sum_{k=-\infty}^{\infty} |h[k]| < \infty .$$

Proof by Contrapositive

Suppose there is an LTI with $h[n]$ such that

$$\sum_{k=-\infty}^{\infty} |h[k]| = \infty .$$

Consider the input $x[n] = \begin{cases} 1 & \text{if } h[-n] > 0 \\ 0 & \text{if } h[-n] = 0 \\ -1 & \text{if } h[-n] < 0 \end{cases}$

Note $|x[n]| \leq 1$ for all n.

Then $y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$

$$y[0] = \sum_{k=-\infty}^{\infty} h[k]\, x[-k]$$

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$\boxed{2.49}$ (b) <u>cont'd</u>

$$y[0] = \sum_{k=-\infty}^{\infty} h[k]\, x[-k]$$

<u>if</u>
$$\begin{cases} h[k] < 0 \Rightarrow x[-k] = -1 \Rightarrow h[k]\, x[-k] = -h[k] > 0 \\ h[k] = 0 \Rightarrow x[-k] = 0 \Rightarrow h[k]\, x[-k] = 0 = h[k] \\ h[k] > 0 \Rightarrow x[k] = 1 \Rightarrow h[k]\, x[-k] = h[k] > 0 \end{cases}$$

~~Thus if any $h[k] < 0$, then~~

$$\cancel{|y[0]| < \sum_{k=-\infty}^{\infty} |h[k]\, x[-k]|}$$

Thus $h[k]\, x[-k] \geq 0$, for all k.

Then $$|y[0]| = \left| \sum_{k=-\infty}^{\infty} h[k]\, x[-k] \right| = \sum_{k=-\infty}^{\infty} |h[k]\, x[-k]|$$

$$|y[0]| = \sum_{k=-\infty}^{\infty} |h[k]|\, |x[-k]| = \sum_{k=-\infty}^{\infty} |h[k]|$$

$\therefore$ $|y[0]| = \infty$ <u>unstable</u> because bounded input gives unbounded output.

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