Homework 1

$\frac{10}{10}$

David Bozarth
CES 543
8·28·06

① 1.3 Total loss = $(5+5+1)$ dB $\qquad P_i = 2\text{mW}$

$$11 \text{ dB} = 10\log\left(\frac{P_i}{P_o}\right) \Rightarrow (10)^{11/10} = \frac{P_i}{P_o}$$

$$P_o = \frac{2\text{mW}}{(10)^{11/10}} = \boxed{0.1589 \text{ mW}}$$ ✓

1.6 RG-19/U has attenuation 22.6 $\frac{\text{dB}}{\text{km}}$ @ 100 MHz.

Total attenuation permissible:

$$10\log\frac{10\text{mW}}{10^{-3}\text{mW}} = 40 \text{ dB}$$

$$\text{Max length} = \frac{40\text{dB}}{22.6\text{ dB}\cdot\text{km}^{-1}} = 1.7699 \text{ km} \rightarrow \boxed{1.77 \text{ km}}$$ ✓

With fiber attenuation 5 $\frac{\text{dB}}{\text{km}}$:

$$\text{Max length} = \frac{40}{5}\text{ km} = \boxed{8 \text{ km}}$$ ✓

1.9 Fiber: # messages = $144(672) = \boxed{96768 \text{ messages}}$

Copper: $900(24) = \boxed{21600 \text{ messages}}$

4.48

It would take $\boxed{5}$ copper cables to exceed the capacity of the one fiber cable.

Using DS-4, the fiber carries up to $144(4032) = 580608$ messages.
It would take $\boxed{27}$ copper cables to exceed this capacity. ✓

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$$\frac{I}{P_i} = 0.65\ A \cdot w^{-1}$$

(1.14) flux $= 10^{10}\ s^{-1}$ $\quad \lambda = 0.8\ \mu m$ $\quad$ Find incident power, and current in detector.

$$P_i = 10^{10}\ hf\ s^{-1} = 10^{10}\ h \cdot \frac{c}{\lambda}\ s^{-1}$$

$$P_i = 10^{10}(2.48)(10^{-19})\ Js^{-1} = \boxed{2.48\ nW}$$ ✓

$$I = 0.65\ nA \cdot nW^{-1}(2.48)\ nW = \boxed{1.61\ nA}$$ ✓

(1.18) $\lambda = 1.06\ \mu m$ $\quad bw = 0.01(f_c)$ $\quad$ Find # voice channels.

A single voice channel occupies 4 KHz.
The carrier frequency is $\frac{c}{\lambda} = \frac{2.99(10^8)\ m \cdot s^{-1}}{1.06(10^{-6})\ m} = 2.82(10^{14})\ Hz$ ✓

The system bandwidth is $2.82(10^{12})\ Hz$

The number of voice channels is $\frac{2.82(10^{12})}{4(10^3)} = \boxed{705 \times 10^6}$ ✓

(1.22) $P_i = 100\ nW$ $\quad$ Let $\Phi \equiv$ # photons per second incident

ⓐ $\lambda = 800\ nm$

$$\Phi = \frac{P_i}{hf} = \frac{\lambda P_i}{ch} = \frac{800(10^{-9})m\ (100)(10^{-9})\ J \cdot s^{-1}}{2.99(10^8)\ m \cdot s^{-1}(6.626)(10^{-34})\ J \cdot s}$$

$$\Phi = \boxed{4.04 \times 10^{11}\ s^{-1}}$$ ✓

ⓑ $\lambda = 1550\ nm$

$$\Phi = \frac{1550}{800}(4.04)(10^{11})\ s^{-1} = \boxed{7.82 \times 10^{11}\ s^{-1}}$$ ✓

ⓒ The longer wavelength requires more photons to deliver a given power (because each photon has lower energy). ✓

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(1.28) System gain $= \left[10. \div (5 + 25 + 15)\right]$ dB $= -35$ dB

System loss $= \boxed{35 \text{ dB}}$ ✓

(1.32) $E = hf = \dfrac{hc}{\lambda}$; $\quad E_{eV} = \dfrac{hc}{\lambda} \cdot \dfrac{1 \text{ eV}}{1.6(10^{-19})\text{ J}}$

$$eV = \frac{6.626(10^{-34})\text{ J}\cdot\text{s}\ (2.99)\text{ m}\cdot\text{s}^{-1}}{1.6(10^{-19})\text{ J}} \cdot \frac{10^{8}}{\boxed{\lambda}\text{ m}}$$

$W_p = \dfrac{1242.3}{\lambda}$ eV where λ is in nm.

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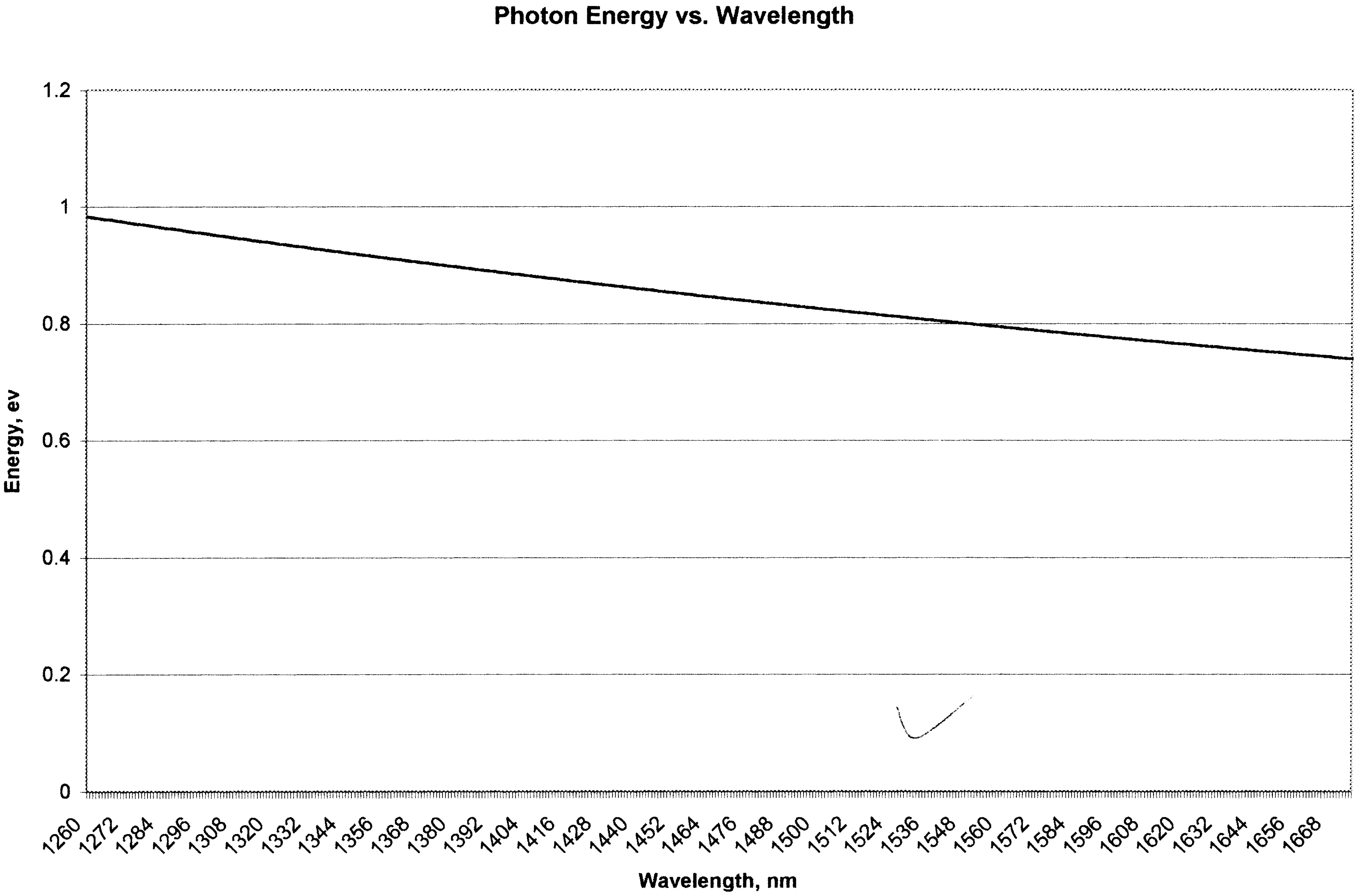
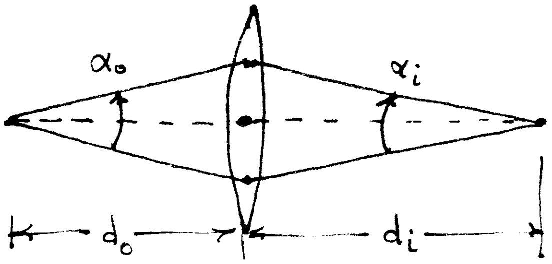

Photon Energy vs. Wavelength
Energy, ev
1.2
1
0.8
0.6
0.4
0.2
0
1260
1272
1284
1296
1308
1320
1332
1344
1356
1368
1380
1392
1404
1416
1428
1440
1452
1464
1476
1488
1500
1512
1524
1536
1548
1560
1572
1584
1596
1608
1620
1632
1644
1656
1668
Wavelength, nm

Homework 2

David Bozarth
CES 543
9.4.06

2.1



$$\frac{\alpha_i}{\alpha_o} = \frac{1}{M} \quad (2.10) \qquad \text{and} \qquad \frac{1}{M} = \frac{d_o}{f} - 1 \qquad (2.7)$$

$$\frac{1}{M} = \frac{d_o}{1}\left(\frac{1}{d_o} + \frac{1}{d_i}\right) - 1 \qquad \left(\begin{matrix}\text{Thin}\\ \text{Lens}\end{matrix}\right)$$

$$\frac{\alpha_i}{\alpha_o} = 1 + \frac{d_o}{d_i} - 1 = \frac{d_o}{d_i} \Rightarrow \boxed{\alpha_i = \alpha_o \cdot \frac{d_o}{d_i}}$$

Let $M = 5$ and $\alpha_o = 40°$. Find α_i.

$$\alpha_i = \frac{\alpha_o}{M} = \boxed{8°}$$

2.2

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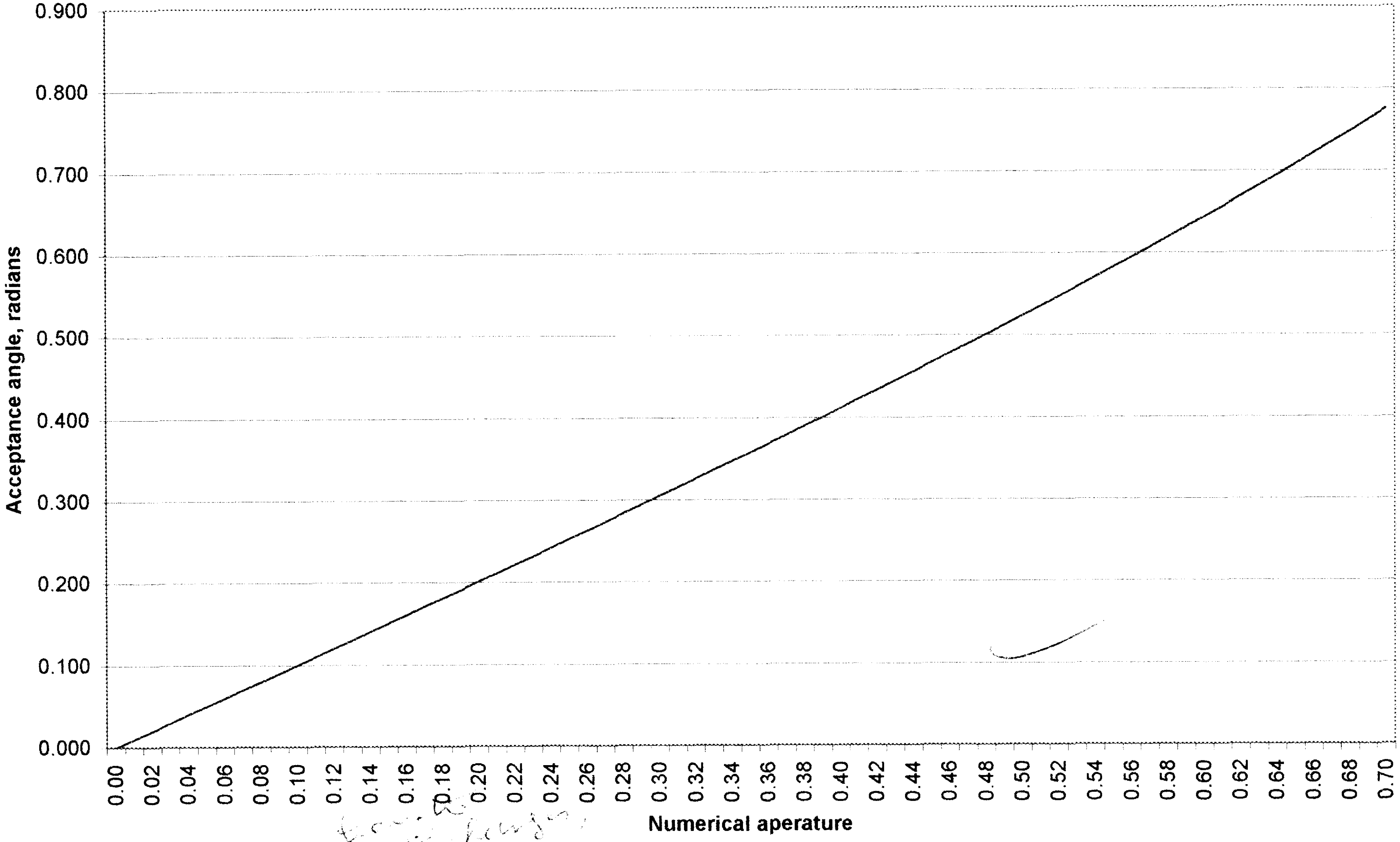
Acceptance angle vs. Numerical Aperature (n = 1.0)



Since you treat
Plot NA. as changing,
N.A. should be on
y-axis !

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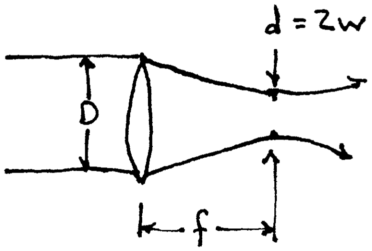
2.5



D = 10 mm , f = 20 mm , $\lambda = 0.8\,\mu m$

Find spot size w at focal plane.

$$w = \frac{2.44}{2} \cdot \frac{\lambda f}{D} = \frac{2.44(0.8)(10^{-6})m\ (20\ mm)}{2 \cdot (10\ mm)}$$

$$w = \boxed{1.952\ \mu m} \quad \text{or} \quad d = 3.9\ \mu m$$

2.12

$$\theta_t = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_i\right) = \sin^{-1}\left(\frac{1.48}{1.46}\sin\theta_i\right)$$

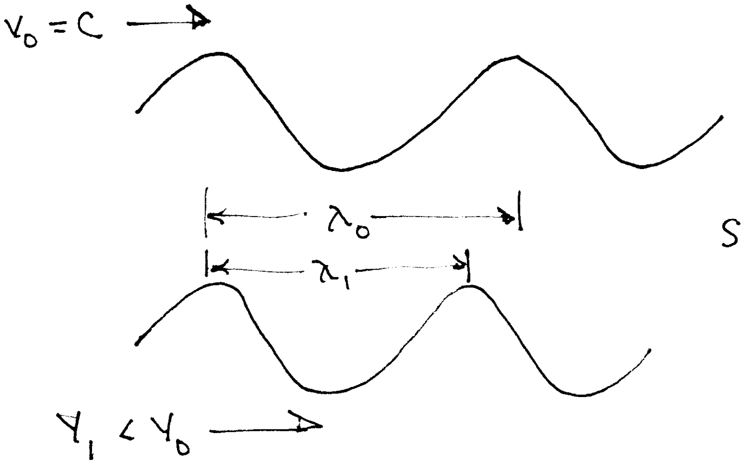
The following graphs show the variation of θ_t with θ_i as θ_i goes from zero° to about 80°.

2.13

Fused silica $n_1 = 1.46$, Silicon $n_2 = 3.5$.

$$\lambda_0 = 800\ nm : \quad v_1 = \frac{c}{1.46} \Rightarrow \lambda_1 = \frac{v_1 \lambda_0}{c} = \frac{\frac{c}{1.46} \cdot 800\,nm}{c}$$

$$f_0 = f_1 = f_2 = \frac{v_0}{\lambda_0} = \frac{v_1}{\lambda_1} = \frac{v_2}{\lambda_2}$$



$$\boxed{\lambda_1 = 548\ nm}$$

Same f, shorter λ, lower v

(Please see the following table)

2.12 cont'd

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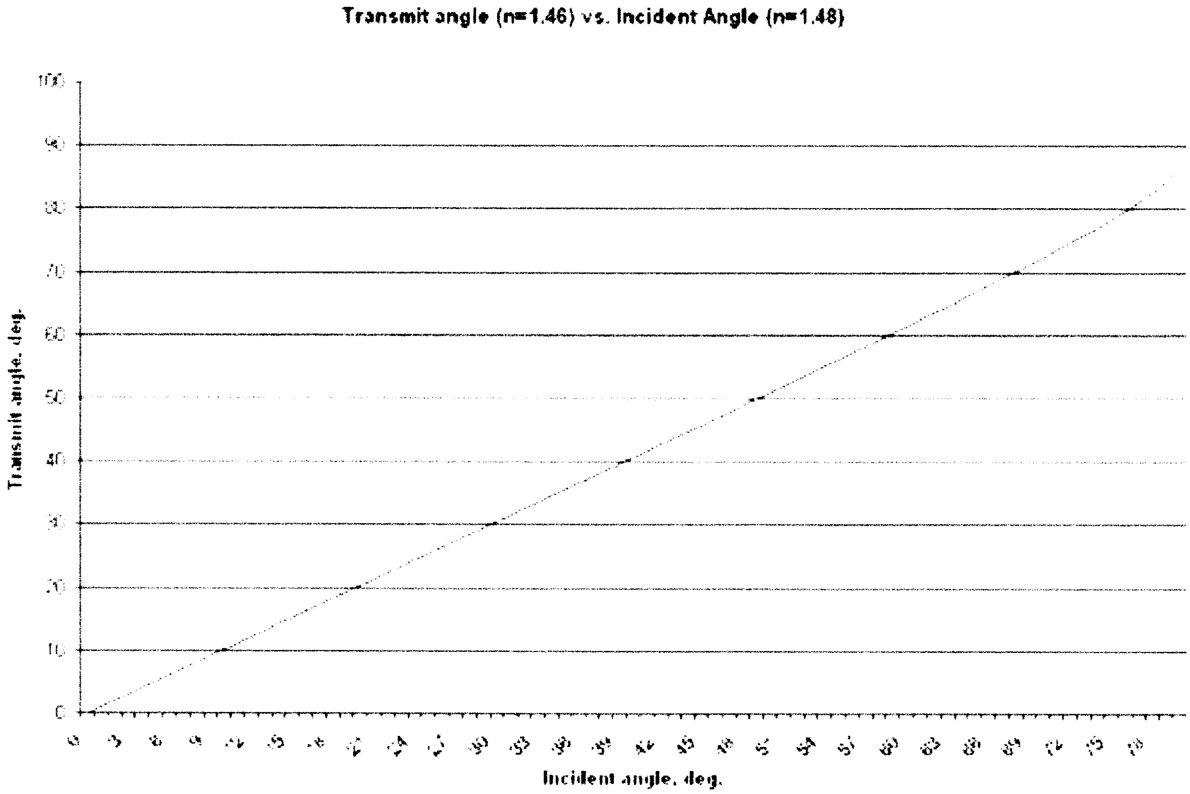
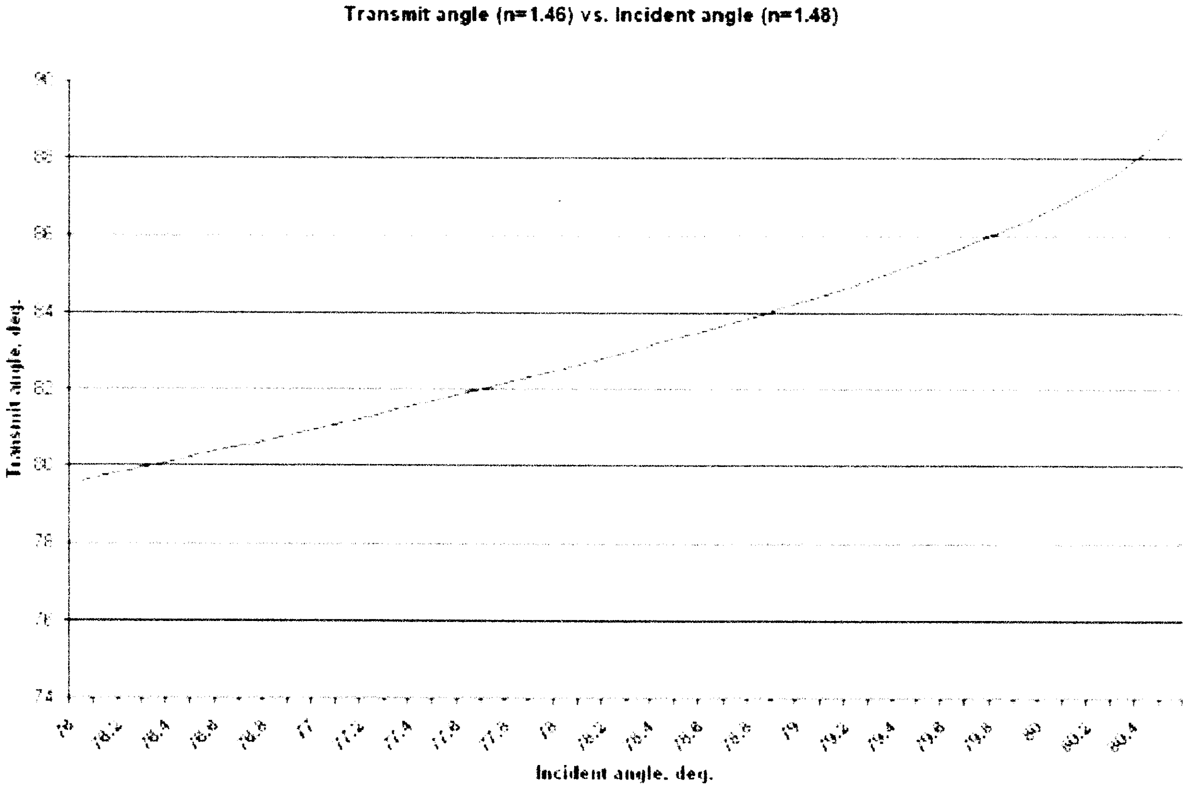

Transmit angle (n=1.46) vs. Incident Angle (n=1.48)

✓


Transmit angle (n=1.46) vs. Incident angle (n=1.48)

✓

2.13 cont'd

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free space		fused silica	silicon
		wavelength, nm	wavelength, nm
wavelength, nm	800	548	229
	1300	890	371
	1550	1062	443

✓

When beam travels from free space into a refractive material, its frequency (and therefore photon energy) remains unchanged.

But the velocity and therefore wavelength are reduced. ✓

Homework 3

10/10

David Bozarth
CES 543
8 Sep 2006

(3.2) $\lambda_0 = 0.85 \mu m$, $\Delta\lambda_1 = 30 nm$, $\Delta\lambda_2 = 2 nm$

Find: pulse spread per unit length $\Delta_1\left(\frac{\tau}{L}\right)$, $\Delta_2\left(\frac{\tau}{L}\right)$

$M = 90 \; ps \cdot nm^{-1} \cdot km^{-1}$ for $\lambda = 0.85 \mu m$

$$\Delta_1\left(\frac{\tau}{L}\right) = -M\Delta\lambda_1 = -90 \; ps \cdot nm^{-1} \cdot km^{-1} (30) \; nm = -2700 \frac{ps}{km}$$

✓

$$\Delta_1\left(\frac{\tau}{L}\right) = \boxed{-2.7 \frac{ns}{km}}$$

$$\Delta_2\left(\frac{\tau}{L}\right) = -90(2) \frac{ps}{km} = \boxed{-180 \frac{ps}{km}} = \boxed{-0.18 \frac{ns}{km}}$$

✓

(3.3) $\lambda_0 = 1.55 \mu m$, $\Delta\lambda_1 = 30 nm$, $\Delta\lambda_2 = 2 nm$

$M = -20 \; ps \cdot (nm \cdot km)^{-1}$ ✓

$$\Delta_1\left(\frac{\tau}{L}\right) = -M\Delta\lambda_1 = (+20)(30) \; ps \cdot km^{-1} = \boxed{0.6 \frac{ns}{km}}$$

$$\Delta_2\left(\frac{\tau}{L}\right) = -M\Delta\lambda_2 = (20)(2) \; ps \cdot km^{-1} = \boxed{0.04 \frac{ns}{km}}$$

✓

(3.4) Using RZ with length 100 m, $\Delta\lambda = 30 nm$, $\lambda_0 = 0.85 \mu m$

$$\underset{\text{electrical}}{bw} = R_{RZ} = \frac{0.35}{\Delta\tau} = \frac{0.35}{\Delta\left(\frac{\tau}{L}\right) \cdot L} = \frac{0.35}{\left|-2.7\right| \frac{ns}{km} (0.1) km} = \boxed{\begin{matrix} 1.3 \; GHz \\ \text{or} \\ 1.3 \; Gbps \end{matrix}}$$

$$R_{NRZ} = 2 R_{RZ} = \boxed{2.6 \; Gbps}$$

See the following table →

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3.4 cont'd

wavelength, μm	line width, nm	distance, km	bw, MHz	Rrz, Mbps	Rnrz, Mbps
0.85	30	0.1	1296	1296	2593
0.85	30	1	130	130	259
0.85	30	10	13	13	26
0.85	2	0.1	19444	19444	38889
0.85	2	1	1944	1944	3889
0.85	2	10	194	194	389
1.55	30	0.1	5833	5833	5833 x 2
1.55	30	1	583	583	583 x 2
1.55	30	10	58	58	58 x 2
1.55	2	0.1	87500	87500	87500 x 2
1.55	2	1	8750	8750	8750 x 2
1.55	2	10	875	875	875 x 2

3.6 source 0.82 μm, line width 1 nm:

$$\frac{\Delta f}{f} = \frac{\Delta\lambda}{\lambda} = \frac{10^{-9}\,m}{0.82(10^{-6})\,m} = 1.22(10^{-3}) = \boxed{0.12\%}$$

$$\Delta f = \frac{c\,\Delta\lambda}{\lambda^2} = \frac{3(10^{8})\,m\cdot s^{-1}\,(10^{-9})\,m}{0.82^2\,(10^{-12})\,m^2} = \boxed{4.46\times10^{11}\,Hz}$$

line width 20 nm:

$$\frac{\Delta f}{f} = \frac{\Delta\lambda}{\lambda} = \frac{20\,(10^{-9})\,m}{0.82\,(10^{-6})\,m} = 2.44\,(10^{-2}) = \boxed{2.4\,\%}$$

$$\Delta f = \frac{c\,\Delta\lambda}{\lambda^2} = \frac{3(10^{8})\,m\cdot s^{-1}\,(20)\,(10^{-9})\,m}{0.82^2\,(10^{-12})\,m^2} = \boxed{8.92\times10^{12}\,Hz}$$

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(3.7) $R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 = \left(\frac{3.6 - 1}{3.6 + 1}\right)^2 = \boxed{0.32}$

~~Loss~~ Gain $(dB) = +10 \log (1 - 0.31947) = \boxed{-1.67 \text{ dB}}$

Loss = 1.67 dB

Homework 4

10/10

David Bozarth
CES 543
9·15·06

(3.10) $E(z) = E_0 e^{-\alpha z}$ where $\alpha = k_0 \sqrt{n_1^2 \sin^2 \theta_i - n_2^2}$

$n_1 = 1.48$
$n_2 = 1.46$

Evanescent field decay with distance from boundary for $\lambda = 0.82\,\mu m$, $0 < z < 4\,\mu m$

note: $k_0 = \frac{2\pi}{\lambda}$

$\theta_i = 84°, 86°, 88°, 90°$

(please see graph ⟶)

3.10 cont'd

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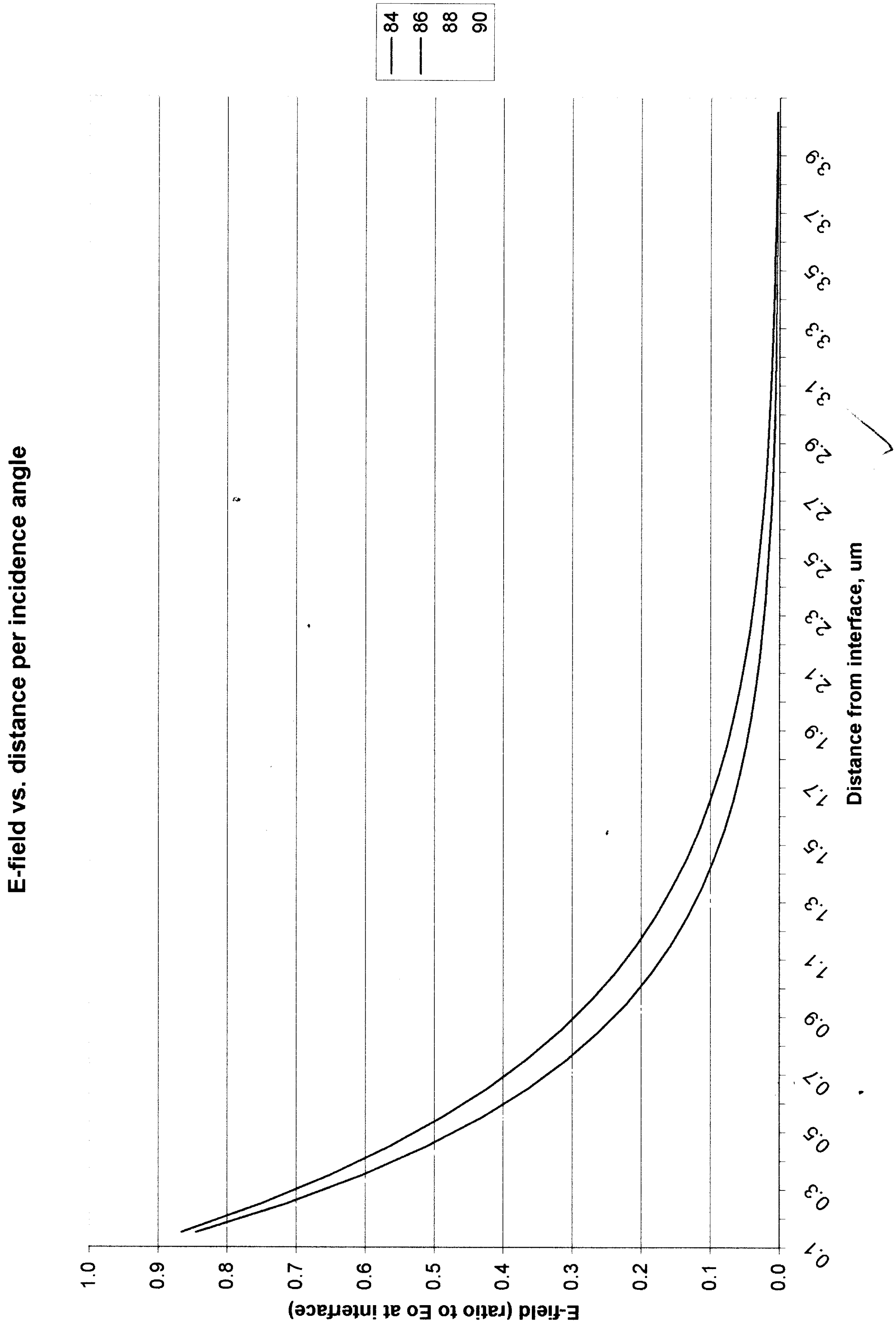

$f_m = 10^3\,s^{-1} \Rightarrow \omega_m = 2000\pi\, s^{-1}$ D. Bozarth

$P_1 = P_{01} + P_{11}\cos(\omega_m t + \phi_1)$ $P_2 = P_{02} + P_{22}\cos(\omega_m t + \phi_2)$

③·12 $P_T = P_1 + P_2 = \boxed{P_{01} + P_{02} + P_{11}\cos(\omega_m t + \phi_1) + P_{22}\cos(\omega_m t + \phi_2)}$

Let $P_{01} = P_{02} = 2\,\mu W$, $P_{11} = P_{22} = 1\,\mu W$

Then $P_T = 4\,\mu W + \cos(2000\pi t + \phi_1) + \cos(2000\pi t + \phi_2)$

Note $\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$.

Let $A = 2000\pi t + \phi_2$, $B = 2000\pi t + \phi_1$

Then $P_T = 4\,\mu W + 2\cos\left(2000\pi t + 2\phi_1\left(\frac{1}{2}\right) + \frac{\phi_2 - \phi_1}{2}\right)\cdot\cos\left[\frac{1}{2}(\phi_2 - \phi_1)\right]\mu W$

Note for any value of $\phi_2 - \phi_1$,

$P_1 = \left[2 + \cos(2000\pi t + \phi_1)\right]\mu W$

$P_2 = \left[2 + \cos(2000\pi t + \phi_2)\right]\mu W$ $\phi_1 + (\phi_2 - \phi_1)$

$\phi_2 - \phi_1 = 0 \Rightarrow P_T = \left[4 + 2\cos(2000\pi t + \phi_1)\right]\mu W$

$P_2 = [2 + \cos(2000\pi t + \phi_1)]\,\mu W$

$\phi_2 - \phi_1 = \frac{\pi}{2} \Rightarrow P_T = \left[4 + \sqrt{2}\cos\left(2000\pi t + \frac{\pi}{4} + \phi_1\right)\right]\mu W$

$P_2 = [2 + \cos(2000\pi t + \phi_1 + \frac{\pi}{2})]\,\mu W$

$\phi_2 - \phi_1 = \pi \Rightarrow P_T = \left[4 + 2\cdot 0\right]\mu W = 4\,\mu W$

$P_2 = [2 + \cos(2000\pi t + \phi_1 + \pi)]\,\mu W$

See graphs ⟶

3.12

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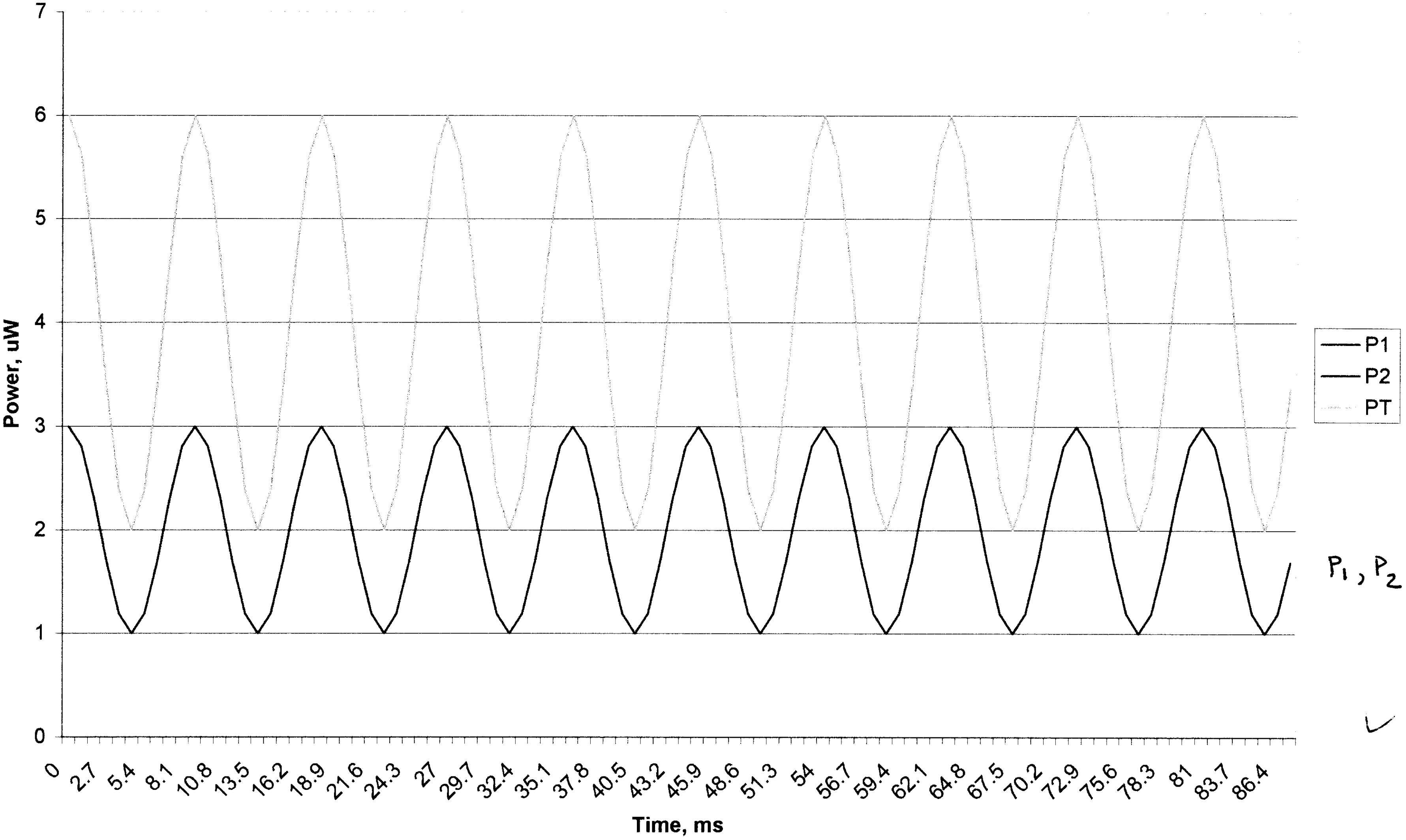

Optical power for 2 wavelengths (in phase), and total power (fm=1 KHz, intensity modulated)

3.12

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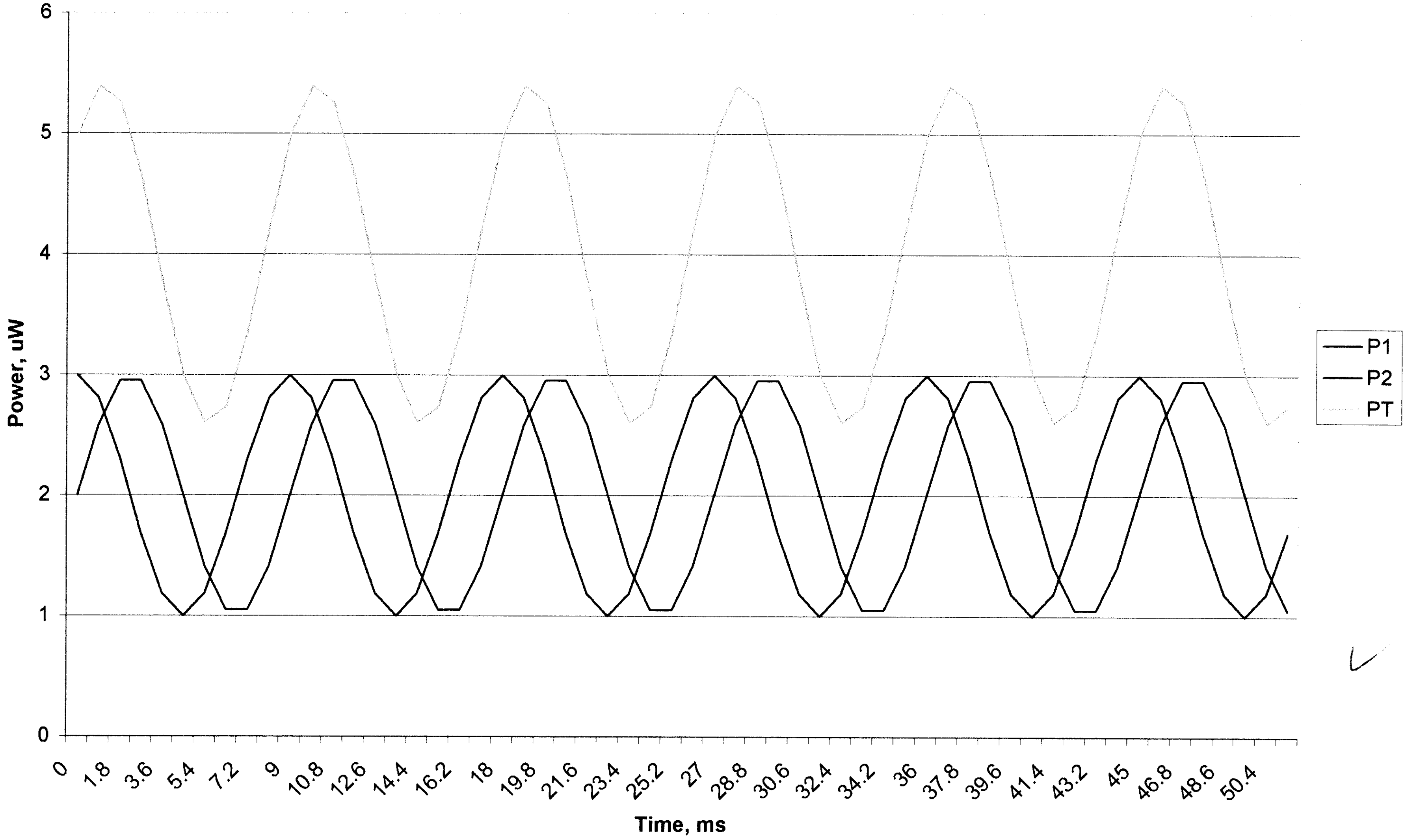
Optical power for 2 wavelengths (delay pi/2), and total power (fm=1 KHz, intensity modulated)



3.12

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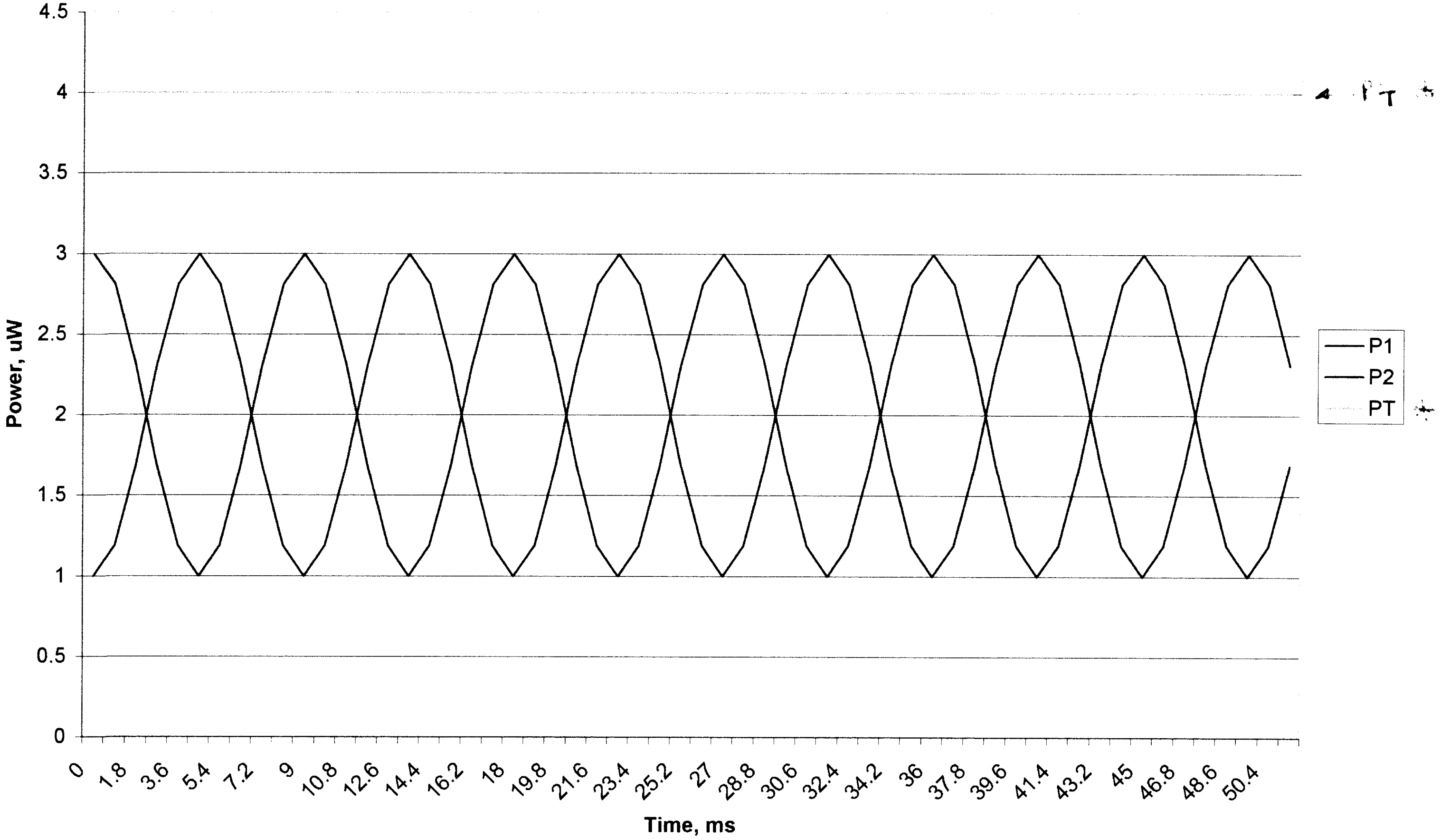
Optical power for 2 wavelengths (delay=pi), and total power (fm=1 KHz, intensity modulated)



$$P_1 + P_2 = 2 + \underbrace{\cos(\omega_m t + \phi_1)} + 2 + \underbrace{\cos(\omega_m t + \phi_1 + \pi)}_{= -\cos(\omega_m t + \phi_1)}$$

$$= 4$$

3.12

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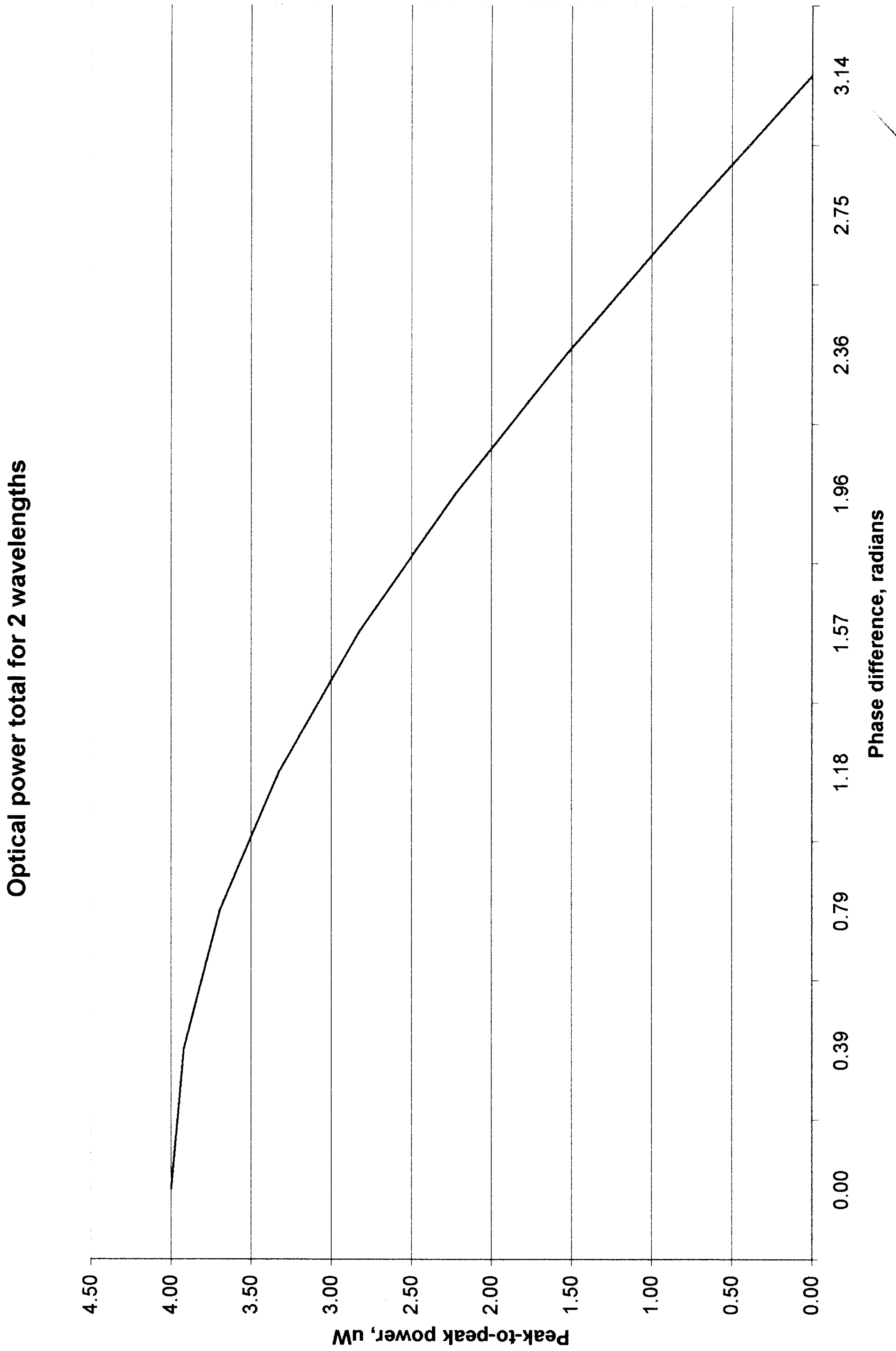

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(3.13) $\theta_i = 85°$ π-polarized $n_1 = 1.48$, $n_2 = 1.465$, $\lambda = 1300\text{nm}$

ⓐ $$\rho_\pi = \frac{-n_2^2 \cos\theta_i + n_1 A}{n_2^2 \cos\theta_i + n_1 A} \quad \text{where } A = \sqrt{n_2^2 - (n_1 \sin\theta_i)^2}$$

$$\underline{\underline{n_1}} A = \overset{j0.2456}{\cancel{j0.1659411}} \rightarrow R9$$

$$n_2^2 \cos\theta_i = 0.1870558 \rightarrow R8$$

$$\rho_\pi = \frac{-0.1871 + \overset{j\,0.2456}{\cancel{j0.1659}}}{0.1871 + j\overset{0.2456}{\cancel{0.1659}}} \cdot \frac{0.1871 - \overset{j\,0.2456}{\cancel{j0.1659}}}{0.1871 - \underset{j\,0.2456}{\cancel{j0.1659}}}$$

$$\rho_\pi = \frac{-0.03499 + \overset{j0.0919}{\cancel{j0.03104}} + \cancel{0.02754}\ 0.0603}{0.03499 + \cancel{j0} + \cancel{0.02754}\ 0.0603}$$

$$\rho_\pi = \cancel{\boxed{\frac{-0.00745 + j0.03104}{0.06253}}} = \frac{0.0253 + j0.0919}{0.0953}$$

$$= 0.2655 + j0.9644 = \boxed{1e^{j74.6°}}$$ ✓

ⓑ Find z for $E(z) = \frac{E(0)}{10} = E_0 e^{-\alpha z} \Rightarrow \frac{1}{10} = e^{-\alpha z}$

$$\Rightarrow z = \frac{\ln 10}{\alpha}$$

where $\alpha = k_0 \sqrt{n_1^2 \sin^2\theta_i - n_2^2}$

$$k_0 = \frac{2\pi}{\lambda} = \frac{2\pi}{1300\text{nm}}$$

$$z = \frac{\ln 10}{\frac{2\pi}{1300\text{nm}} \sqrt{1.48^2 \sin^2(85°) - 1.465^2}}$$

$$z = \boxed{2.87\ \mu\text{m}}$$ ✓

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(3.17) $L_f = 6 \text{ dB}$ for $f_m = 2 \text{ GHz}$ **Find:** $bw_{elec} = f_{3dB}\,(elec)$

$$6 \text{ dB} = -10 \log \left\{ \exp\left[-0.693 \left(\frac{f_m}{f_{3dB}} \right)^2 \right] \right\}$$

$$-\frac{3}{5} = \log \left\{ \exp\left[-0.693 \left(\frac{2(10^9)\, s^{-1}}{f_{3dB\,(opt)}} \right)^2 \right] \right\}$$

$$10^{-3/5} = \exp\left[-0.693\,(4)\,(10^{18})\, s^{-2} \cdot \frac{1}{(f_{3dB\,(opt)})^2} \right]$$

$$\ln\left(10^{-3/5}\right) = -4\,(0.693)\,(10^{18})\, s^{-2} \left(\frac{1}{f_{3dB\,(opt)}} \right)^2$$

$$f_{3dB\,(opt)} = \left(\frac{-4\,(0.693)\,(10^{18})\, s^{-2}}{\ln\left(10^{-3/5}\right)} \right)^{1/2}$$

$$f_{3dB\,(elec)} = (0.71)\, f_{3dB\,(opt)} = \boxed{1.01 \text{ GHz}}$$ ✓

(3.20) $\max \Delta\left(\frac{\tau}{L}\right) = 3 \text{ ps} \cdot \text{km}^{-1}$, $\Delta\lambda = 2 \text{ nm}$

Find: max ~~$\Delta\lambda$~~ $|\lambda - \lambda_0|$ where λ_0 is zero-dispersion
λ is operating point

$$M = \frac{-0.095 \text{ ps} \cdot \text{nm}^{-2} \cdot \text{km}^{-1}}{4} \left(\lambda - \frac{\lambda_0^4}{\lambda^3} \right)$$ units of $\frac{\text{ps}}{\text{nm} \cdot \text{km}}$

This is specified on p.70 for $\lambda_0 = 1300 \text{ nm}$

cont'd

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(3.20) $\Delta\left(\frac{\tau}{L}\right) = -M\,\Delta\lambda \Rightarrow M = -\frac{3}{2}\ \text{ps}\cdot\text{nm}^{-1}\cdot\text{km}^{-1}$

Then $-\frac{3}{2} = -\frac{0.095}{4}\left(\lambda - \frac{1300^4}{\lambda^3}\right)$ where λ in nm. (see pp. 70-71)

$$\frac{2(3)}{0.095} = \lambda - \frac{1300^4}{\lambda^3}$$

$$0 = \lambda - \frac{6}{0.095} - \frac{1300^4}{\lambda^3}$$

$$0 = \lambda^4 - \frac{6}{0.095}\lambda^3 - 1300^4$$ ✓

Find the root:

Newton's method didn't work using initial approximations of 1300 and 1500 (separately).

Using the Secant Method with initial approximations of 1200 and 1400 (together), I found a ~~the~~ root to be $13\underline{16}.0818476859$.

So $\boxed{\Delta\lambda = 16 \text{ nm}}$

Using the chart on p. 69 is hopeless because the resolution is too low.

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(3.22) Given $M(\lambda) = \frac{M_0}{4}\left(\lambda - \frac{\lambda_0^4}{\lambda^3}\right)$ Dispersion about $\lambda = \lambda_0$

<u>Prove</u>: M_0 is slope of dispersion curve at $\lambda = \lambda_0$.

<u>Proof</u>: $M(\lambda) = \frac{M_0}{4}\lambda - \left(\frac{M_0}{4}\lambda_0^4\right)\frac{1}{\lambda^3}$

$$\frac{dM}{d\lambda} = \frac{M_0}{4} - \frac{M_0}{4}\lambda_0^4(-3)\lambda^{-4} = \frac{M_0}{4}\left(1 + 3\frac{\lambda_0^4}{\lambda^4}\right)$$ ✓

$$\left.\frac{dM}{d\lambda}\right|_{\lambda=\lambda_0} = M_0 \qquad \#$$

✓

D. Bozarth
CES 543
10.5.06

$\boxed{\text{Homework 5}}$

④.① AlGaAs $\lambda = 0.82\,\mu m$ $\theta = 85^\circ$

Plot peak E amplitude vs y. Find d and n_{eff}.

In film: $$E = \underset{\substack{\uparrow \\ \text{constant}}}{E_1} \cos hy \underbrace{\sin(\omega t - \beta z)}_{\substack{\text{constant} \\ \text{for given } y}}$$

Outside film: $$E = \underset{\substack{\uparrow \\ \text{constant}}}{E_2}\, e^{-\alpha(y-\frac{d}{2})} \underbrace{\sin(\omega t - \beta z)}_{\text{constant}}$$

Where E_2 is max value of E-field at film ed

$$\alpha = \frac{2\pi}{\lambda}\sqrt{n_1^2 \sin^2\theta - n_2^2}$$

See graph →

Find d:

$$\tan\left(\frac{hd}{2}\right) = \frac{1}{n_1 \cos\theta}\sqrt{n_1^2\sin^2\theta - n_2^2} = 1.622172$$

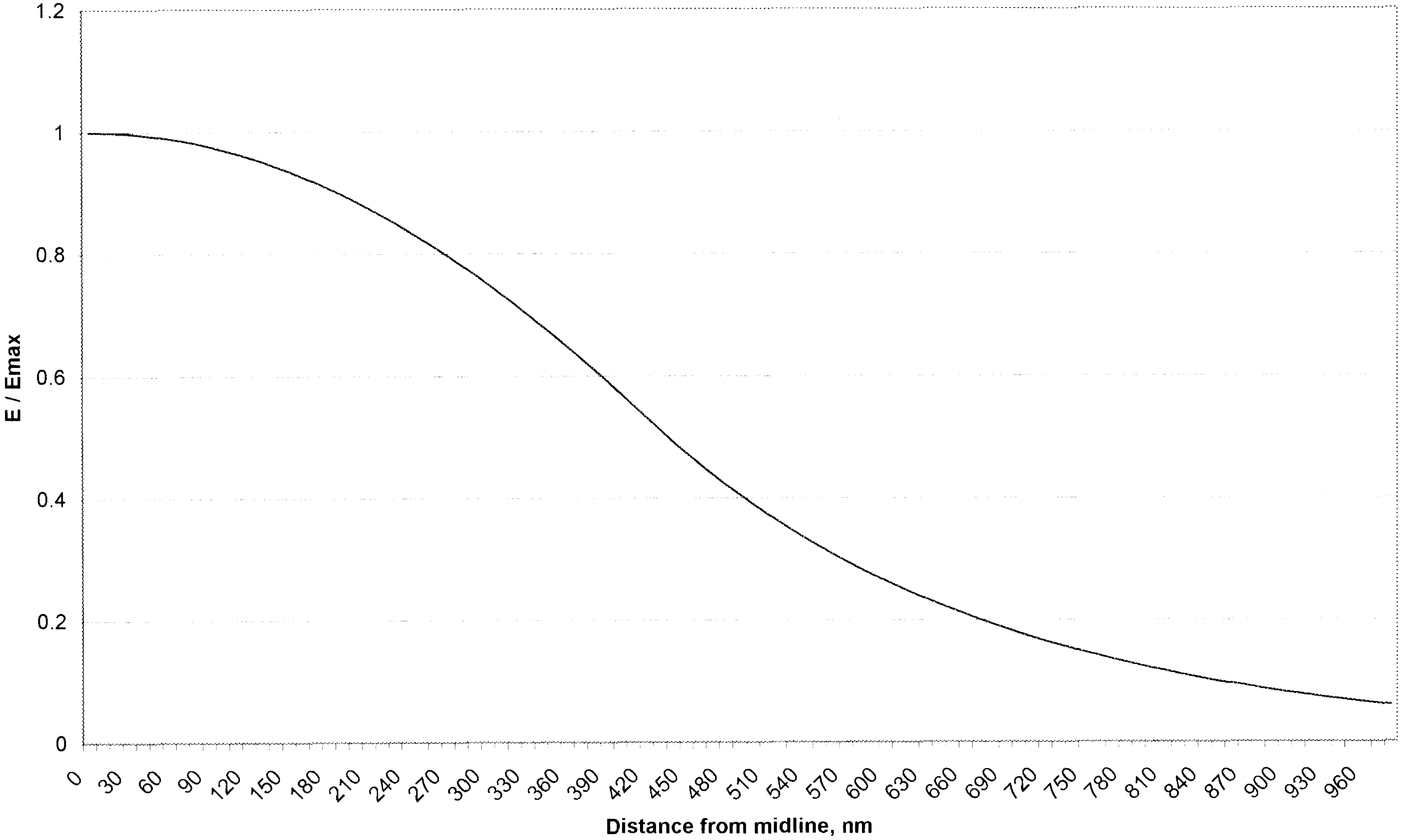
$$hd = 2\tan^{-1}(1.622172) = 116.6959^\circ = 2.03672$$
radi

$$\frac{d}{\lambda} = \frac{hd}{2\pi n_1 \cos\theta} = 1.033129 \Rightarrow \boxed{d = 0.8472\,\mu m}$$

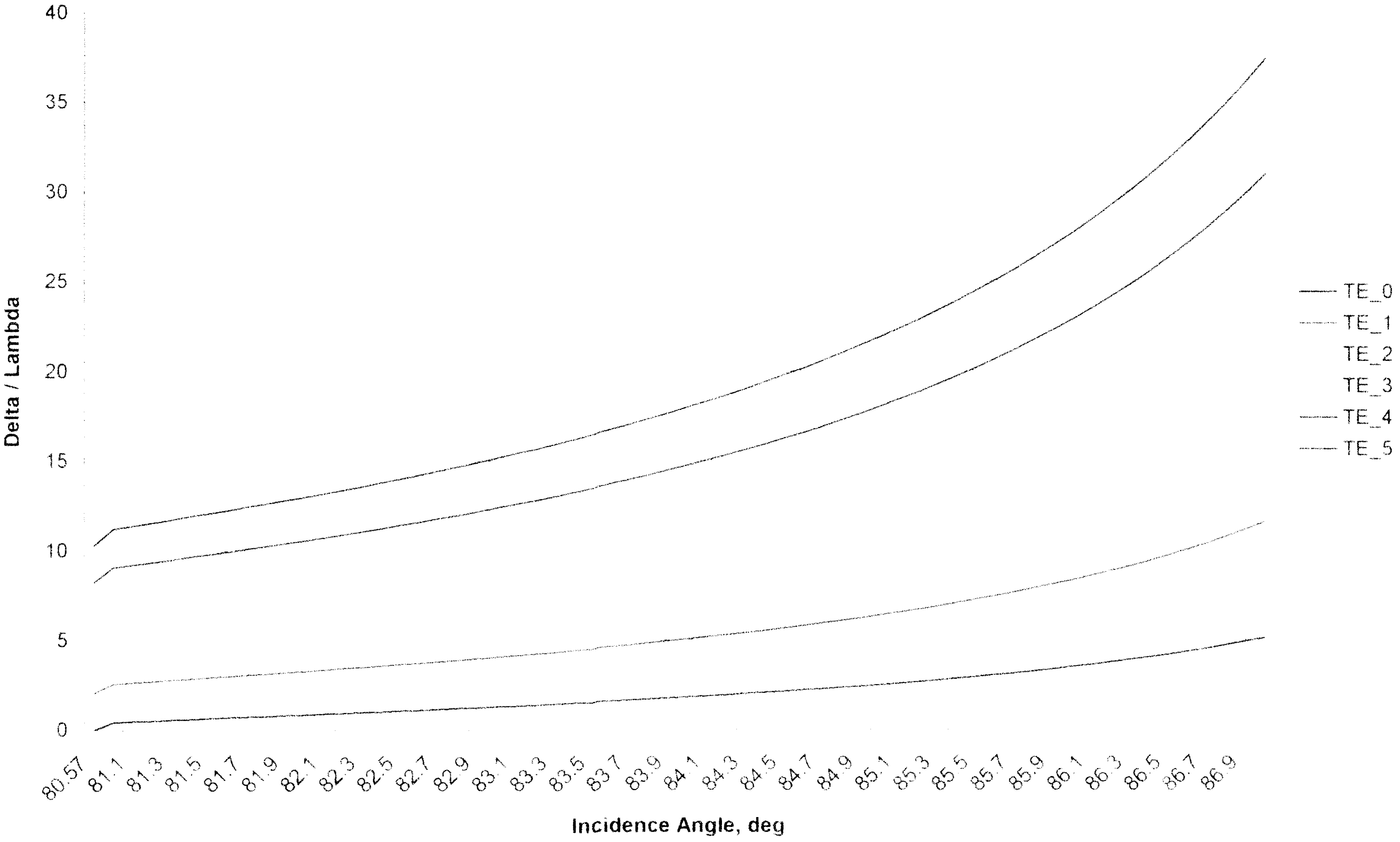
Find n_{eff}:

$$n_{eff} = n_1 \sin\theta \Rightarrow \boxed{n_{eff} = 3.59}$$

Transverse E-field



TE Mode Chart (n1=1.48, n2=1.46)



D. Bozarth

(4.4) AlGaAs, symmetrical slab $n_1 = 3.6$, $n_2 = 3.55$

How many $\overset{N}{\text{modes}}$ can propagate for $\frac{d}{\lambda} = 5, 10, 100$

$\frac{d}{\lambda} = 5 \Rightarrow$ highest mode $m = 5(2)\sqrt{n_1^2 - n_2^2}$

$m = 5.97$

$\boxed{N = 6}$

$\frac{d}{\lambda} = 10 \Rightarrow m = 20\sqrt{n_1^2 - n_2^2} = 11.96$

$\boxed{N = 12}$

$\frac{d}{\lambda} = 100 \Rightarrow m = 200\sqrt{n_1^2 - n_2^2} = 119.6$

$\boxed{N = 120}$

D. Bozarth

(4.7) **Prove:** The condition for cutoff for the mth TE mod is

$$\left(\frac{d}{\lambda}\right)_{m,c} = \frac{m}{2\sqrt{n_1^2 - n_2^2}}$$

Proof:

Cutoff occurs when $\theta = \theta_c \Rightarrow n_1 \sin^2\theta - n_2^2 = 0$

(Eq. 4.11) Thus, if m is even, $\tan \frac{hd}{2} = 0$ at cutoff.

~~If m is odd, $\tan\left(\frac{hd}{2} - \frac{\pi}{2}\right) = 0$ at cutoff.~~ True but not neede

Taking periodicity of the tangent function into account,

$$\left(\frac{d}{\lambda}\right)_m = \left(\frac{d}{\lambda}\right)_0 + \frac{m}{2n_1 \cos\theta} = \quad \text{(Eq. 4.12)}$$

$$\left(\frac{d}{\lambda}\right)_m = \frac{hd}{2\pi n_1 \cos\theta} + \frac{m}{2n_1 \cos\theta} \quad \text{(p. 98)}$$

$$= \frac{1}{2n_1 \cos\theta}\left(\frac{hd}{\pi} + m\right)$$

Suppose $m = 0$. At cutoff, $\frac{hd}{2} = 0$

$$\Rightarrow hd = 0$$

$$\Rightarrow \left(\frac{d}{\lambda}\right)_{,c} = \frac{m}{2n_1 \cos\theta} = \frac{m}{2\sqrt{n_1^2 - n_2^2}}$$

D. Bozarth

4.7 cont'd Suppose for some $k > 0$,

$$\left(\frac{d}{\lambda}\right)_{k,c} = \frac{k}{2\sqrt{n_1^2 - n_2^2}}$$

Per Eq. 4.13, $\Delta\left(\frac{d}{\lambda}\right) = \frac{1}{2 n_1 \cos\theta} = \frac{1}{2\sqrt{n_1^2 - n_2^2}}$

Thus $\left(\frac{d}{\lambda}\right)_{k+1,c} = \frac{k}{2\sqrt{n_1^2 - n_2^2}} + \frac{1}{2\sqrt{n_1^2 - n_2^2}}$

$$\left(\frac{d}{\lambda}\right)_{k+1,c} = \frac{k+1}{2\sqrt{n_1^2 - n_2^2}}$$

By induction we see that

$$\left(\frac{d}{\lambda}\right)_{m,c} = \frac{m}{2\sqrt{n_1^2 - n_2^2}} \qquad \#$$

D. Bozarth

4.8 **Show**: $n_1 \approx n_2 \Rightarrow NA \approx n_1 \sqrt{2\Delta}$

where $\Delta = \frac{n_1 - n_2}{n_1}$

$$NA = \sqrt{n_1^2 - n_2^2} \approx \sqrt{n_1^2 - n_1 n_2} = \sqrt{n_1(n_1 - n_2}$$

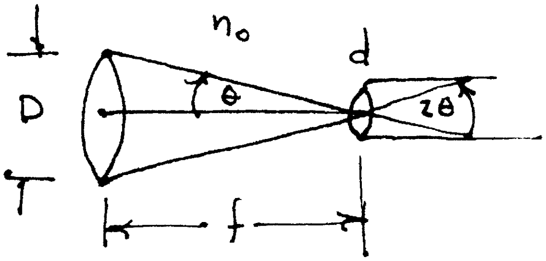
$$n_1 \sqrt{2\Delta} = \sqrt{2 n_1^2 \Delta} = \sqrt{2 n_1^2 \left(\frac{1}{n_1}\right)(n_1 - n_2)} = \sqrt{2 n_1 (n_1 -}$$

So the approximated value of NA differs from $n_1 \sqrt{2\Delta}$ by a factor of $\sqrt{2}$.

D. Bozarth

(4.10) AlGaAs $\lambda = 0.82\ \mu m$ $\frac{d}{\lambda} = 10$ Gaussian beam dia = 1 mm = D spot

Design the coupling lens (focal length)



$NA = \sqrt{n_1^2 - n_2^2} = 0.5979$

Suppose $n_o = 1$

Then $n_o \sin\theta = 0.5979$

$\theta = 36.72^\circ$

Focus to a point (minimum f)

$$\tan\theta = \frac{D}{2f} \Rightarrow f = \frac{D}{2\tan\theta} = \frac{10^{-3}\ m}{2\tan\theta} = \boxed{\begin{array}{c}0.67\ mm\\ \min f\end{array}}$$

Also want the focused Gaussian spot radius w to obey

$$w_o = \frac{\lambda f}{\pi W} \quad \text{where } W = \text{beam radius}$$

$$f = \frac{W w_o \pi}{\lambda} = \frac{\pi(0.5)(10^{-3})m\ (4.1)(10^{-6})m}{0.82(10^{-6})\ m} = \boxed{\begin{array}{c}7.85\ mm\\ \max f\end{array}}$$

So $\boxed{0.67\ mm < f < 7.85\ mm}$