

Homework 5

Problem 1: Let Q = probability of one message failing. Let $P(n)$ be the probability of the n th message succeeding. Then

$$P(n) = (1 - Q) Q^{(n - 1)}$$

Let N be the number of messages that must be transmitted in order to get one good message. The expected value of N is:

$$E(N) = \sum n P(n) = \sum n (1 - Q) Q^{(n - 1)} = (1 - Q) \sum n Q^{(n - 1)}$$

$$E(N) = (1 - Q) / (1 - Q)^2 = 1 / (1 - Q)$$

$$\text{Since } Q = (1 - 0.8^{10}) = 0.8926258, \text{ then } E(N) = 9.31 \dots$$

Therefore, **10 transmissions** must be sent in order to expect that, on the average, one good transmission will occur.

Another way to obtain the same result *in this case* (since Q constant for every message):

$n P' \geq 1$ where P' represents the probability of one message succeeding.

Since the transmissions are all independent,

$$n \geq 1 / P' = 1 / 0.8^{10} = 9.31 \rightarrow \mathbf{10 \text{ transmissions}}$$
 must be sent.

Problem 5:

The original string:

0111101111101111110

The new string with stuffed bits & locations shown clearly:

01111011111 0 011111 0 10

The new string actually transmitted:

011110111110011111010

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Problem 15:

$M(x)$ is represented by 10011101

$G(x)$ is represented by 1001

Append 4 zeros to $M(x) \rightarrow x^4 M(x) = 100111010000$

Divide $G(x)$ into $x^4 M(x) \rightarrow 100111010000 / 1001 = 100011001$ with $R(x) = 0001$

Now subtract $x^4 M(x) - R(x) = \mathbf{100111010001} = \mathbf{T(x)}$.. the Transmit string.

During transmission, the 3rd bit from left is inverted $\rightarrow 101111010001$

On receipt, this string is divided by $G(x)$:

$$101111010001 / 1001 = 101010001 \text{ with } R(x) = 1000$$

Since the remainder is nonzero, we have detected a transmission error.

Problem 32:

$$\#bits = 1.544 (10^6) \text{ bit s}^{-1} (1 / ((2/3) (2.998) (10^{10}) \text{ cm s}^{-1})) (100 \text{ km}) (10^2 \text{ cm}/10^{-3} \text{ km})$$

$$\#bits = 772.515 \rightarrow \mathbf{772 \text{ bits}}$$